FUNCTIONS AND INVERSES NOTES 22/02/2026

Definition of a function: A function f, is defined as a relationship between values, where each input value maps to one output value. (In other words: for an equation to be called a function there can be only one y-value for a particular x-value)

There are two types of functions:

1. One-to-one function – it is a function where there is a single y-value for a particular x-value
2. Many-to-one function – A function cannot have more than one y value to each x-value. However, a function can have more than one x-value for a particular y-value.

Vertical line test:

To test if a graph is a function, you can use the vertical line test: If a vertical line (a line parallel to the y-axis) touches the graph more than once at any point, the graph is not a function

ARE THE FOLLOWING FUNCTIONS OR NON-FUNCTIONS

a) (1 ; -2) ; (5 ; 3) ; (15 ; -2) ; (6; 8) ---- Function
b) (11 ; 12) ; (51 ; 3) ; (15 ; -2) ; (51; 8) ---- Non Function
c) (1 ; -2) ; (5 ; 3) ; (15 ; 12) ; (6; 8) --- -Function

❖ **IF ALL *X*-VALUES ARE DIFFERENT IT IS A FUNCTION**
THE *Y* -VALUES CAN REPEAT ITSELF

EXAMPLE:
For what values of a and b is the following a function:
(a; 3) ; (13 ; 7) ; (-5 ; 9) ; (7 ; b)

Solution:
$a \in R$; $a \neq \{13 ; -5 ; 7\}$
$b \in R$;

1. LINES OF SYMMETRY
RULES

- ***x- axis*** $(x ; y) \rightarrow (x ; -y)$
- ***y-axis*** $(x ; y) \rightarrow (-x ; y)$
- ***y = x*** $(x ; y) \rightarrow (y ; x)$
- ***y = – x*** $(x ; y) \rightarrow (-y ; -x)$
- **ORIGIN** $(x ; y) \rightarrow (-x ; -y)$

EXAMPLE:

Given: $f(x) = 2x - 6$

a) Determine $f^{-1}(x)$

b) **Sketch both** $f(x)$ and $f^{-1}(x)$ on the same set of axis.

c) **Determine the point of intersection of** f *and* f^{-1}

Solution:

$a)\ f : y = 2x - 6$

$f^{-1:} : x = 2y - 6$

$2y = x + 6$

$y = \frac{1}{2}x + 3$

$f^{-1}(x) = \frac{1}{2}x + 3$

$b) f : y = 2x - 6$

X-intercept Put $y = 0$

$2x - 6 = 0$

$x = 3$

Y- intercept Put $x = 0$

$y = 2(0) - 6$

$y = -6$

$f^{-1}(x) = \frac{1}{2}x + 3$

X-intercept Put Y= 0

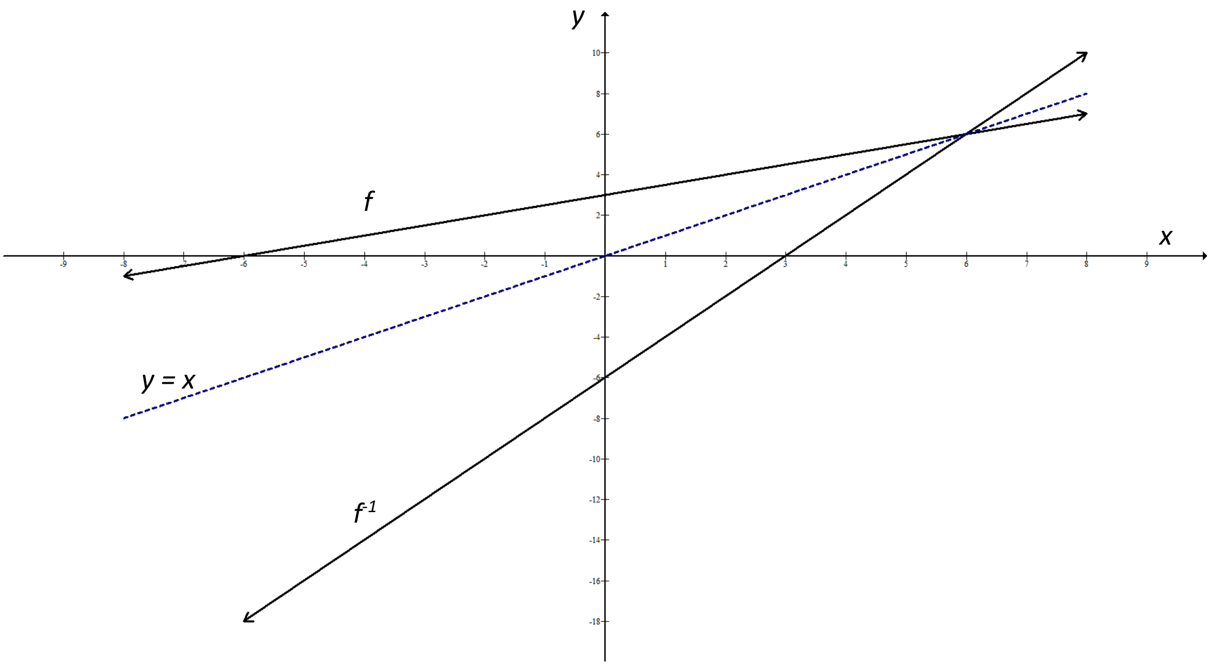
$\frac{1}{2}x + 3 = 0$

$x = -6$

Y- intercept Put $x = 0$

$y = \frac{1}{2}(0) + 3$

$y = 3$



$c)\quad y = 2x - 6.....(1)$

$y = x.............(2)$

substitute (1) into (2)

$2x - 6 = x$

$x = 6$

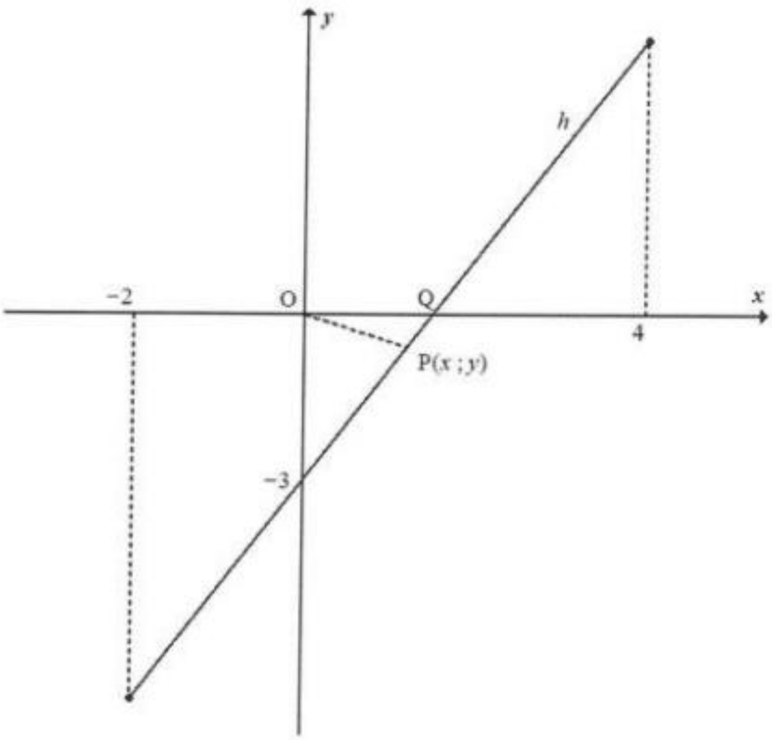
$y = 6$

$(6:6)$

FUNCTIONS AND INVERSES

QUESTION 5

Given: $h(x) = 2x - 3$ for $-2 \leq x \leq 4$. The x-intercept of h is Q.



5.1 Determine the coordinates of Q. (2)

5.2 Write down the domain of h^{-1}. (3)

5.3 Sketch the graph of h^{-1} in your ANSWER BOOK, clearly indicating the y-intercept and the end points. (3)

5.4 For which value(s) of x will $h(x) = h^{-1}(x)$? (3)

5.5 P$(x\,;\,y)$ is the point on the graph of h that is closest to the origin. Calculate the distance OP. (5)

5.6 Given: $h(x) = f'(x)$ where f is a function defined for $-2 \leq x \leq 4$.

5.6.1 Explain why f has a local minimum. (2)

5.6.2 Write down the value of the maximum gradient of the tangent to the graph of f. (1)

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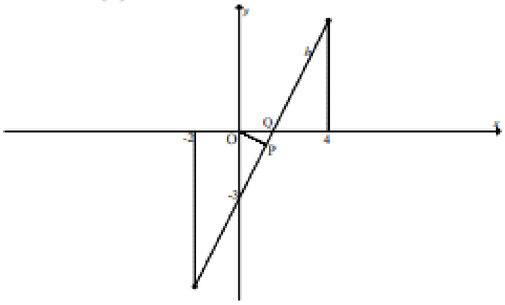
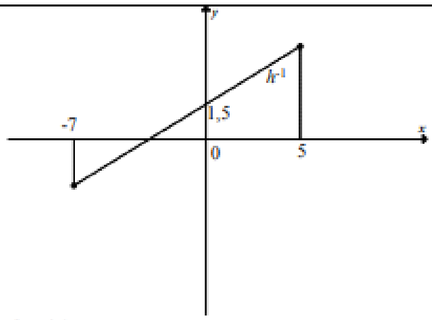
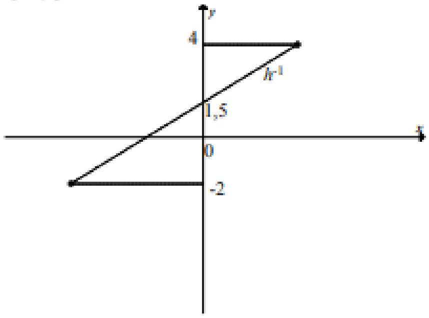
QUESTION/*VRAAG* 5

	Given $h(x)=2x-3$ for $-2\leq x\leq 4$. 	
5.1	For x-intercepts, $y=0$ $2x-3=0$ $x=1{,}5$ $Q(1{,}5\ ;\ 0)$	✓ $x=1{,}5$ ✓ $y=0$ (2)
5.2	h: $x=-2$: $y=2(-2)-3=-7$ $x=4$: $y=2(4)-3=5$ Domain of h^{-1}: $-7\leq x\leq 5$ **OR/*OF*** $[-7;\ 5]$	✓ $h(-2)=-7$ ✓ $h(4)=5$ ✓ $-7\leq x\leq 5$ (3)
5.3	 **OR/*OF*** 	✓ y-intercept on a straight line ✓ line segment ✓ accurate endpoints (x or y or both) (3)

5.4	$h(x)=2x-3$ For the inverse of *h*, $x=2y-3$ $y=\frac{x+3}{2}$ $h^{-1}(x)=\frac{x+3}{2}$ $h(x)=h^{-1}(x)$ $2x-3=\frac{x+3}{2}$ $4x-6=x+3$ $x=3$	✓ $y=\frac{x+3}{2}$ ✓ $2x-3=\frac{x+3}{2}$ ✓ $x=3$ (3)
	OR/*OF* $h(x)=2x-3$ *h* and h^{-1} intersect when $y=x$ $h(x)=x$ $2x-3=x$ $x=3$	✓ $h(x)=x$ ✓ $2x-3=x$ ✓ $x=3$ (3)
	OR/*OF* $h(x)=2x-3$ For the inverse of *h*, $x=2y-3$ $y=\frac{x+3}{2}$ $h^{-1}(x)=x$ $\frac{x+3}{2}=x$ $x+3=2x$ $x=3$	✓ $y=\frac{x+3}{2}$ ✓ $\frac{x+3}{2}=x$ ✓ $x=3$ (3)

Parabola

EXAMPLE 2

Determine the equation of the inverse of $y = 3x^2$.

Solution

Original: $y = 3x^2$

Inverse: $x = 3y^2$

$\therefore y^2 = \frac{x}{3}$

$\therefore y = \pm\sqrt{\frac{x}{3}}$

Note: This inverse gives two outputs for each input. This means that the inverse is **not a function**. More about this later.

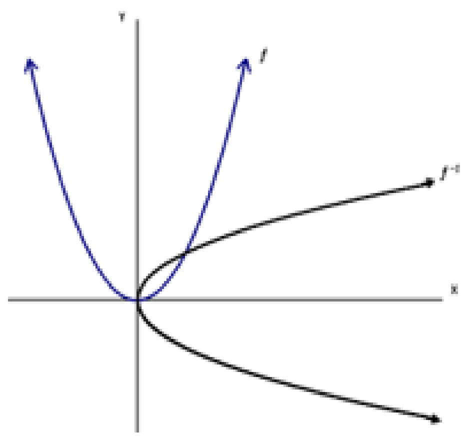
b) Sketch both f(x) = $3x^2$ and its inverse on the same set of axes.

$f(x) = 3x^2$

x	–1	0	1
y	3	0	3

$f(x)^{-1} = \sqrt{\frac{x}{3}}$

x	3	0	3
y	–1	0	1



EXAMPLE 7

Given the function $y=-2x^2$.

(a) Determine the equation of the inverse of this function.

(b) Sketch the graphs of $y=-2x^2$ and its inverse on the same set of axes and show the line of symmetry.

(c) Determine the coordinates of the points of intersection between $y=-2x^2$ and its inverse.

Solution

(a) Original: $y=-2x^2$

Inverse: $x=-2y^2$

$\therefore y^2=\frac{x}{-2}$

$\therefore y=\pm\sqrt{-\frac{x}{2}}$

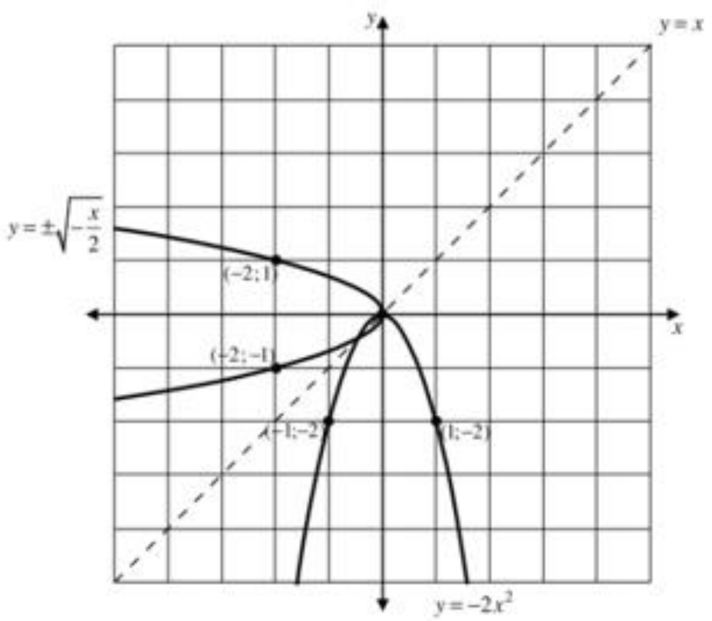
(b) $y=-2x^2$ is a parabola with a negative orientation.

Three points on $y=-2x^2$:

$(-1;-2)$
$(0;0)$
$(1;-2)$

Invert coordinates for $y=\pm\sqrt{-\frac{x}{2}}$:

$(-2;-1)$
$(0;0)$
$(-2;1)$



(c) Solve $y=-2x^2$ and $y=x$ simultaneously:

$-2x^2=x$

$\therefore 2x^2+x=0$

$\therefore x(2x+1)=0$

$\therefore x=0$ or $x=-\frac{1}{2}$

Points of intersection: $(0;0)$ and $\left(-\frac{1}{2};-\frac{1}{2}\right)$

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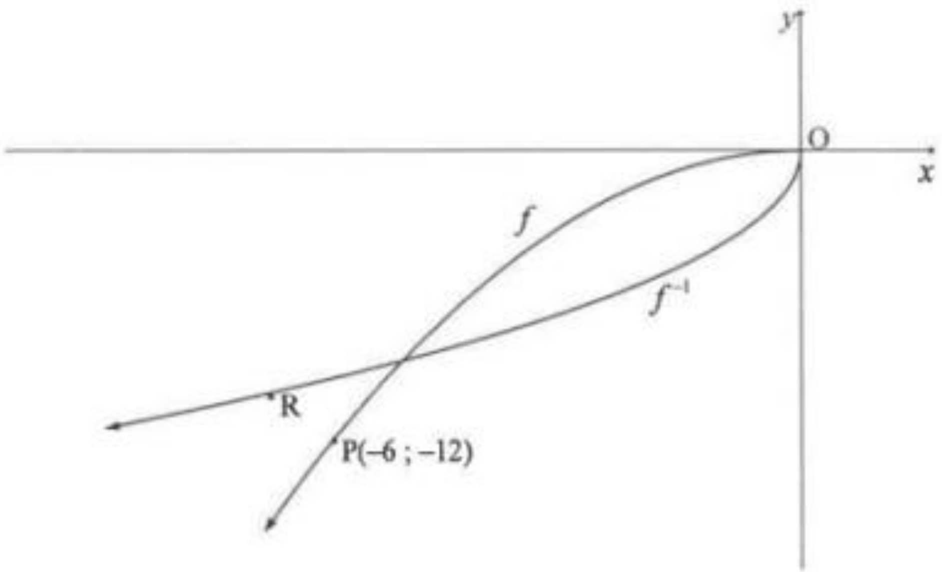
QUESTION 4

In the diagram below, the graph of $f(x)=ax^2$ is drawn in the interval $x \leq 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.



4.1	Is f^{-1} a function? Motivate your answer.	(2)
4.2	If R is the reflection of P in the line $y=x$, write down the coordinates of R.	(1)
4.3	Calculate the value of a.	(2)
4.4	Write down the equation of f^{-1} in the form $y=...$	(3)
		[8]