MONYETLA NOTES SEQUENCES AND SERIES 14/02/2026

June 2017:

QUESTION 3

Given the quadratic sequence: 0; 17; 32; ...

3.1 Determine an expression for the general term, T_n, of the quadratic sequence. (4)

3.2 Which terms in the quadratic sequence have a value of 56? (3)

3.3 Hence, or otherwise, calculate the value of $\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$. (4)

[11]

June 2017:

Q3.1

First differences: 17; 15

Second difference: –2

$$T_n = an^2 + bn + c$$

$$a = \frac{\text{second difference}}{2} = -1$$

$$3a + b = 17$$

$$3(-1) + b = 17$$

$$b = 20$$

$$a + b + c = 0$$

$$-1 + 20 + c = 0$$

$$c = -19$$

$$T_n = -n^2 + 20n - 19$$

Q3.2

$$56 = -n^2 + 20n - 19$$

$$n^2 - 20n + 75 = 0$$

$$(n - 15)(n - 5) = 0$$

$$n = 5 \text{ or } n = 15$$

Q3.3

$$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$$

$= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T$

$= (T_5 - T_{15}) + (T_6 - T_{14}) + \ldots + (T_9 - T_{11}) + T_{10}$

$= T_{10}$

because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ ...

$T_{10} = -(10)^2 + 20(10) - 19$

$= 81$

Nov 2019

QUESTION 3

3.1 Without using a calculator, determine the value of: $\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$ (3)

Nov 2019

Q3.1

$$\sum_{y=3}^{10} \frac{1}{y-2} - \sum_{y=3}^{10} \frac{1}{y-1}$$

$$= \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{8}\right) - \left(\frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{8} + \frac{1}{9}\right)$$

$$= 1 - \frac{1}{9}$$

$$= \frac{8}{9}$$

EXAMPLE:

1.1 Write in $\sum$ notation ie. $\sum$ general term

3 ; 7 ; 11 ; to 20 terms

$$a = 3 \quad d = 4$$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 3 + (n-1)4 \\ &= 3 + 4n - 4 \\ &= 4n - 1 \end{aligned}$$

$$\sum_{k=1}^{20} 4k - 1$$

1.2 Write in $\sum$ notation

3 ; 6 ; 12 to 20 terms

$$a = 3; \quad r = 2$$

$$\begin{aligned} T_n &= ar^{n-1} \\ &= 3(2)^{n-1} \end{aligned}$$

$$\sum_{k=1}^{20} 3(2)^{k-1}$$

1.3 Hence write the following in $\sum$ notation

$$1 ; \frac{7}{6} ; \frac{11}{12}$$

$$\frac{3}{3} ; \frac{7}{6} ; \frac{11}{12}$$

$$\sum_{k=1}^{20} \frac{4k-1}{3(2)^{k-1}}$$

March 2017:

3.2 Determine the value(s) of x in the interval $x \in [0^\circ ; 90^\circ]$ for which the sequence -1 ; $2\sin 3x$; 5 ; ..... will be arithmetic. (4)

Q3.2

$$-1 ; 2\sin 3x ; 5; ...$$

$$2\sin 3x + 1 = 5 - 2\sin 3x$$

$$4\sin 3x = 4$$

$$\sin 3x = 1$$

$$3x = 90^\circ$$

$$x = 30^\circ$$

May-June 2019

2.2 Given a geometric sequence: 36 ; -18 ; 9 ; ...

2.2.1 Determine the value of r, the common ratio. (1)

2.2.2 Calculate n if $T_n = \frac{9}{4\,096}$ (3)

2.2.3 Calculate S_∞ (2)

2.2.4 Calculate the value of $\frac{T_1 + T_3 + T_5 + T_7 + ... + T_{499}}{T_2 + T_4 + T_6 + T_8 + ... + T_{500}}$ (4)

[17]

MAY /JUNE 2019

Q2.2.1

$$r = \frac{-18}{36} = -\frac{1}{2}$$

Q2.2.2

$$T_n = 36\left(-\frac{1}{2}\right)^{n-1}$$

$$\frac{9}{4096} = 36\left(-\frac{1}{2}\right)^{n-1}$$

$$\frac{1}{16384} = \left(-\frac{1}{2}\right)^{n-1}$$

$$\left(-\frac{1}{2}\right)^{14} = \left(-\frac{1}{2}\right)^{n-1}$$

$$14 = n-1$$

$$n = 15$$

Q2.2.3

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{36}{1-\left(-\frac{1}{2}\right)}$$

$$= 24$$

Q2.2.4

$$\frac{T_1 + T_3 + T_5 + T_7 + \ldots + T_{499}}{T_2 + T_4 + T_6 + T_8 + \ldots + T_{500}}$$

$$= \frac{a + ar^2 + ar^4 + \ldots + ar^{498}}{ar + ar^3 + ar^5 + \ldots + ar^{499}}$$

$$= \frac{a + ar^2 + ar^4 + \ldots + ar^{498}}{r(a + ar^2 + ar^4 + \ldots + ar^{498})}$$

$$= \frac{1}{r}$$

QUESTION 2

2.1 Given the arithmetic series: $5 + 7 + 9 + \ldots + 93$

2.1.1 Determine the general term of the series, T_n, in the form $T_n = pn + q$. (2)

2.1.2 The given series represents the number of kilometres that an athlete ran each week in preparation for an ultramarathon. The athlete ran 93 km in the last week of the training programme. How long, in weeks, was the training programme? (2)

2.1.3 The athlete used this opportunity to raise funds for her high school. The community sponsored her R10 for each kilometre run during the training programme. Calculate the total amount that the athlete raised for her school. (3)

QUESTION/*VRAAG* 2

2.1.1	$T_n = 2n + 3$	✓$2n$ ✓3 (2)
2.1.2	$93 = 2n + 3$ $90 = 2n$ $45 = n$	✓equating ✓answer (2)
2.1.3	$50 + 70 + 90 + ... + 930$ $S_{45} = \frac{45}{2}[2(50) + (45 - 1)(20)]$ $S_{45} = R22050$ Total raised = R22 050 **OR/*OF*** $50 + 70 + 90 + ... + 930$ $S_{45} = \frac{45}{2}[50 + 930]$ $S_{45} = R22050$ Total raised = R22 050 **OR/*OF*** $5 + 7 + 9 + ... + 93$ $S_{45} = \frac{45}{2}[2(5) + (45 - 1)(2)]$ $S_{45} = 2\ 205km$ $S_{45} = R22050$ Total raised = R22 050 **OR/*OF*** $5 + 7 + 9 + ... + 93$ $S_{45} = \frac{45}{2}[5 + 93]$ $S_{45} = 2\ 205km$ $S_{45} = R22050$ Total raised = R22 050	✓convert to money ✓substitution ✓answer (3) **OR/*OF*** ✓convert to money ✓substitution ✓answer (3) **OR/*OF*** ✓substitution ✓answer ✓convert to money (3) **OR/*OF*** ✓substitution ✓answer ✓convert to money (3)

2.2 The general term of a geometric sequence is $T_n = 2^{n+2}$

2.2.1 Write down:

(a) The first term (1)

(b) The common ratio (1)

2.2.2 Calculate T_{20} (Write your answer as a power of 4.) (2)

2.2.3 Calculate $\sum_{n=1}^{\infty} \frac{1}{T_n}$ (3)

2.2.4 Consider the first 21 terms of the sequence $T_n = 2^{n+2}$. Calculate the sum of the terms in this sequence that are not powers of 4. (4)

[18]

2.2.1 a)	$T_1 = (2)^{1+2}$ $T_1 = 8$	✓$a = 8$ (1)
2.2.1 b)	$r = 2$	✓$r = 2$ (1)
2.2.2	$T_{20} = (2)^{20+2}$ $T_{20} = 2^{22} = (2^2)^{11}$ $= 4^{11}$	✓substitution ✓answer (2)

2.2.3	$\frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$ $S_\infty = \frac{a}{1-r}$ $= \frac{\frac{1}{8}}{1-\frac{1}{2}}$ $\therefore S_\infty = \frac{1}{4}$	✓series ✓substitution ✓answer (3)
2.2.4	$8 + 4^2 + 32 + 4^3 + \ldots + 4^{11} + \ldots$ $S_{21} - S_{10} = \frac{8(2^{21}-1)}{2-1} - \frac{16(4^{10}-1)}{4-1}$ $= 16\ 777\ 208 - 5\ 592\ 400$ $= 11\ 184\ 808$ **OR/*OF*** $8 + 32 + 128 +$ $S_{11} = \frac{8(4^{11}-1)}{4-1}$ $\therefore S_{11} = 11\ 184\ 808$	✓$\frac{8(2^{21}-1)}{2-1}$ ✓$n = 10$ ✓$\frac{16(4^{10}-1)}{4-1}$ ✓11 184 808 (4) **OR/*OF*** ✓$n = 11$ ✓$r = 4$ ✓$\frac{8(4^{11}-1)}{4-1}$ ✓11 184 808 (4)
		[18]

QUESTION 3

Given the quadratic sequence: 14 ; 9 ; 6 ; 5 ; …

3.1 Show that the general term of this sequence is $T_n = n^2 - 8n + 21$. (3)

3.2 Two consecutive terms of the quadratic sequence have a difference of 33. Calculate the value of the larger term. (3)

3.3 The value of m is added to each term in the quadratic sequence. Determine the values of m for which only the terms between T_1 and T_7 of the quadratic sequence will have negative values. (3)

[9]

QUESTION/*VRAAG* 3

3.1	14 ; 9 ; 6 ; 5 ; …. -5 -3 -1 2 2 $2a=2$ $3(1)+b=-5$ $1-8+c=14$ $a=1$ $b=-8$ $c=21$ $\therefore T_n=n^2-8n+21$	✓ $2a=2$ ✓ $3(1)+b=-5$ ✓ $1-8+c=14$ (3)
3.2	$T_n=-5+(n-1)(2)$ $T_n=2n-7$ $2n-7=33$ $\therefore n=20$ $\therefore T_{21}=(21)^2-8(21)+21$ $T_{21}=294$ **OR/*OF*** $\therefore T_{n+1}-T_n=(n+1)^2-8(n+1)+21-n^2+8n-21$ $n^2+2n+1-8n-8+21-n^2+8n-21=33$ $2n-7=33$ $\therefore n=20$ $\therefore T_{21}=(21)^2-8(21)+21$ $T_{21}=294$	✓general term ✓equating to 33 ✓answer (3) **OR/*OF*** ✓ $T_{n+1}-T_n$ ✓equating to 33 ✓answer (3)
3.3	$T_7=T_1=14$ $\therefore 14+m\geq 0$ $m\geq -14$ And $T_6=T_2$ $\therefore 9+m<0$ $m<-9$ $\therefore -14\leq m<-9$	✓ -14 ✓ -9 ✓ $-14\leq m<-9$ (3)
		[9]