TOPIC 6: HYDRAULICS

Prescribed content

Hydraulics (fluid mechanics) is a topic in applied science and engineering dealing with mechanical properties of liquids.

Thrust

- Define thrust as the normal force exerted by a liquid at rest on a given surface in contact with it.

Pressure

- Pressure at a particular point is the thrust acting on the unit area around that point.

$$\text{pressure} = \frac{\text{thrust}}{\text{area}} \quad \therefore \quad p = \frac{F}{A}$$

SI unit: $N \cdot m^{-2}$ OR pascal (Pa)

- Use the above equation to calculate pressure, thrust and area.

Practical unit of pressure

- 1 atmosphere = $1{,}01 \times 10^5$ Pa
- 1 bar = 10^5 Pa = 100 kPa
- 1 torr = 133 Pa

Fluid pressure

- Fluid pressure is given by the following equation:

$$p = \rho g h$$

- Use the above equation to calculate fluid pressure, thrust, height and density.

Pascal's law

Pascal's law states that in a continuous liquid at equilibrium, the pressure applied at any point is transmitted equally to other parts of the liquid.

Hydraulic lift

- The hydraulic lift is used to lift heavy loads.
- Discuss the use of hydraulic lifts in technology.
  Examples: car lifts and jacks, hydraulic brakes, dentist chairs
- In hydraulic lifts:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \qquad A_2 > A_1$$

- Use the above equation to calculate force, area and radius of the pistons.

YouTube Videos

Number	Duration	Topic
1	4:16	Laws of liquid pressure
2	4:12	Application of Pascal's law
3	1:34	Hydraulic lift
4	4:21	Hydraulic press

Important terms/definitions

Barometer	A scientific instrument used to measure atmospheric pressure.
Density Symbol: rho (ρ)	The mass per unit volume of a substance.
Fluid	A fluid is a state of matter such as a liquid or a gas that deforms (flows) under a shear force and conforms to the shape of the container.
Hydraulics (fluid mechanics)	A topic in applied sciences and engineering dealing with mechanical properties of liquids.
Hydraulic lift	Machinery used to lift heavy objects like cars.
Manometer	An instrument for measuring the pressure of a fluid, consisting of a tube filled with a liquid, the level of the liquid being determined by the fluid pressure and the height of the liquid being indicated on a scale.
Pascal's law	In a continuous liquid at equilibrium, the pressure applied at any point is transmitted equally to other parts of the liquid.
Pressure	Pressure at a particular point id the thrust acting on the unit area around that point.
Thrust	The normal force exerted by a liquid at rest on any given surface with which it is in contact with.

Formulae

$p = \frac{F}{A}$	p: pressure in pascal (Pa) F: thrust (force) in newton (N) A: area in square metres (m^2)
$p = \rho gh$	p: pressure in pascal (Pa) ρ: density in kilogram per cubic metre ($kg \cdot m^{-3}$) g: gravitational acceleration in metres per square second ($m \cdot s^{-2}$) h: height in metres (m)
$\rho = \frac{m}{V}$	ρ: density in kilogram per cubic metre ($kg \cdot m^{-3}$) - SI unit gram per cubic centimetre ($g \cdot cm^{-3}$) m: mass in gram (g) or kilogram (kg) v: volume in cubic centimeter (cm^3) or cubic metre (m^3)
$A = \lambda \times b$	A: area of a rectangle in square metres (m^2) ℓ: length in metre (m) b: breadth in metre (m)
$A = \pi r^2$ $A = \frac{\pi d^2}{4}$	A: area of a circle in square metres (m^2) r: radius in metre (m) d: diameter in metre (m)
$V = \ell bh$	V: volume of a rectangle in cubic metres (m^3) ℓ: length in metre (m) b: breadth in metre (m) h: height in metre (m)
$V = \pi r^2 h$	V: volume of a cylinder in cubic metres (m^3) r: radius in metre (m) h: height in metre (m)
$\frac{F_1}{A_1} = \frac{F_2}{A_2}$	F_1: thrust exerted on area 1 in newton (N) F_2: thrust exerted on area 2 in newton (N) A_1: area of surface 1 in square metres (m^2) A_2: area of surface 2 in square metres (m^2)

Metric Conversion

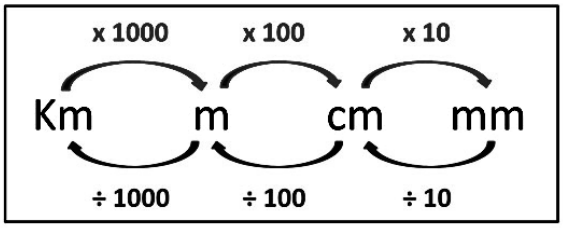
King	**H**enry	**D**ied	**U**nusually	**D**rinking	**C**hocolate	**M**ilk
Kilo	Hecto	Deca	* Unit *	Deci	Centi	Milli
 **10 x 10 x 10 x LARGER than a unit**	**10 x 10 x LARGER than a unit**	**10 x LARGER than a unit**	***Meter*** *(length)* ***Liter*** *(liquid volume)* ***Gram*** *(mass/weight)* **1 unit**	**10 x SMALLER than a unit**	**10 x 10 x SMALLER than a unit**	**10 x 10 x 10 x SMALLER than a unit** 
1 kilo = 1,000 units	1 hecto = 100 units	1 deca = 10 units		10 deci = 1 unit	100 centi = 1 unit	1,000 milli = 1 unit
km = kilometer kL = kiloliter kg = kilogram	hm = hectometer hL = hectoliter hg = hectogram	dam = decameter daL = decaliter dag = decagram	m = meter L = liter g = gram	dm = decimeter dL = deciliter dg = decigram	cm = centimeter cL = centiliter cg = centigram	mm = millimeter mL = milliliter mg = milligram
Example: 5 kilo	50 hecto	500 deca	5,000 units	50,000 deci	500,000 centi	5,000,000 milli

DIVIDE numbers by 10 if you are getting bigger (same as moving decimal point one space to the left)

MULTIPLY numbers by 10 if you are getting smaller (same as moving decimal point one space to the right)

Converting LENGTH Units

It is easiest to use a conversion look-up diagram like the one below.

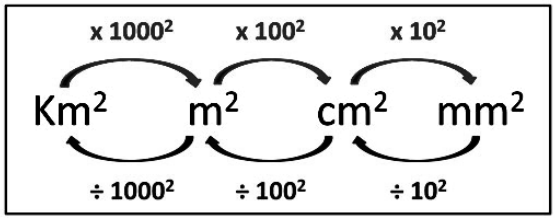


5km = ? m **Need to x 1000** 5 x 1000 = **5000m** ✓

120cm = ? m **Need to ÷ 100** 120 ÷ 100 = **1.2m** ✓

Converting AREA Units

AREA consists of Square Units, so we need to SQUARE all our Lengths.



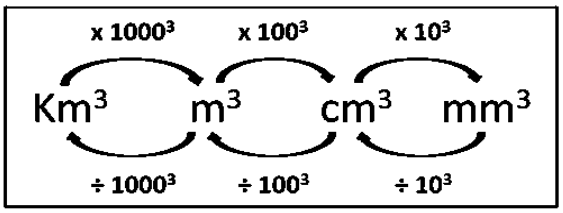
5km² = ? m² **Need to x 1000²** 5 x 1000 x 1000 = **5 000 000 m²** ✓

1200cm² = ? m² **Need to ÷ 100²** 1200 ÷ 100 ÷ 100 = **0.12 m²** ✓

Converting VOLUME Units

VOLUME is how much 3D space is occupied, and is measured in cubes.

VOLUME consists of Cube Units, so we need to CUBE all our Lengths.



VOLUME conversions use powers of 3, and usually create very large results.

3m³ = ? cm³ **Need to x 100³** 3 x 100 x 100 x 100 = **3 000 000 cm³** ✓

Hydraulics

Mechanical properties of liquids

Pressure

Thrust exerted per unit area

$$p = \frac{F}{A}$$

SI: pascal (Pa) or newton per square metre ($N \cdot m^{-2}$)

	Other units			
	pascal (Pa)	**bar (bar)**	**torr (Torr)**	**atmosphere (atm)**
1 Pa	1	1×10^{-5}	$7,5 \times 10^{-3}$	$9,9 \times 10^{-6}$
1 bar	1×10^{5}	1	750	0,99
1 Torr	133	133×10^{-5}	1	$1,3 \times 10^{-3}$
1 atm	$1,01 \times 10^{5}$	1,01	≈ 760	1

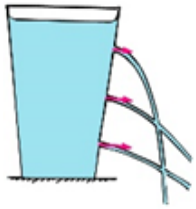
Fluid pressure

A fluid exerts pressure as a result of its weight.

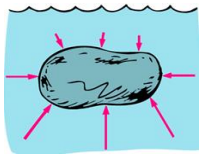
$$p = \rho g h$$

1. Increases with density

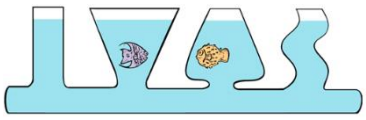
2. Increases with depth



3. Is exerted in all directions



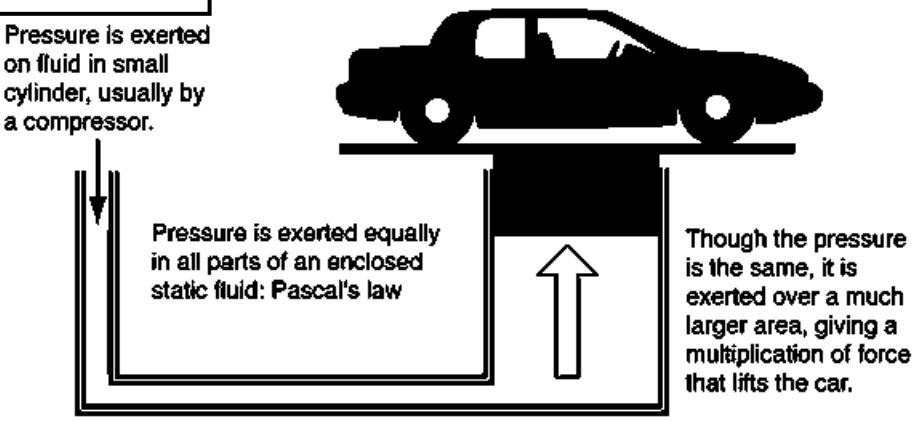
4. Is independent of size and shape of the container



Pascal's law

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Applied in hydraulic lift



The force in the small cylinder must be exerted over a much larger distance. A small force exerted over a large distance is traded for a large force over a small distance.

6.1 Pressure

Example 1

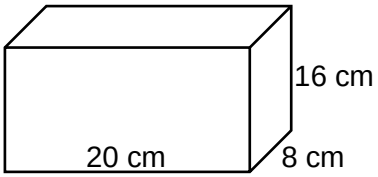
Calculate the pressure exerted by an object of mass 20 kg on an area of 0,2 m^2.

Answer

$p = \frac{F}{A} = \frac{mg}{A} = \frac{20 \times 9,8}{0,2} = 980$ Pa

Example 2

Lead has a density of 11,34 $g \cdot cm^{-3}$. A lead block 20 cm long, 8 cm wide and 16 cm high is placed on a desk. Calculate the pressure exerted by the block on the desk.



Answer

Use the density of the block and its volume to calculate its mass.

$V = (20)(8)(16) = 2\,560\ cm^3$

$\rho = \frac{m}{V} \therefore 11,34 = \frac{m}{2560} \therefore m = 29\,030,4\ g = 29,0304\ kg$

$p = \frac{F}{A} = \frac{mg}{A} = \frac{29,0304 \times 9,8}{0,2 \times 0,08} = 17\,7810,12$ Pa

Activity 6.1: Pressure

1. The SI unit of pressure is:

   A ohms B pascal C joule D watt

2. The unit of pressure one bar is:

   A 1 Pa B 1 kPa C 100 kPa D 1000 kPa

3. In symbols, pressure is equal to …

   A $\frac{A}{F}$ where A is the area and F is the force.
   B $\frac{F}{A}$ where F is the force and A is the area.
   C $\frac{d}{F}$ where d is the distance and F is the force.
   D $\frac{F}{d}$ where F is the force and d is the distance.

4. Convert 300 Torr to:

   4.1 Pa 4.2 bar 4.3 atmosphere

5. A girl of mass 40 kg wears heels with an area of 1 cm^2 in contact with ground. Calculate the pressure exerted by the girl on the ground. (*Answer: 3,92 x 10^6 Pa*)

6. A small table weighing 40 N stands on four legs, each having an area of 0,001 m^2. Calculate the pressure that the table exerts on the floor. *(Answer: 10 000 Pa)*

7. A thrust of 450 N is exerted on a certain area. If the pressure on the area is 4 500 Pa, calculate the size of the area. *(Answer: 0,1 m^2)*

8. An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank reads 6,90 x 10^6 Pa. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0,150 m in diameter? *(Answer: 1,22 x 10^5 N)*

9. A car weighing 1,2 x 10^4 N rests on four tyres. If the gauge pressure in each tyre is 200 kPa, what is the area of the tyres in contact with the road? *(Answer: 0,015 m^2)*

6.2 Fluid pressure

Example 3
Calculate the pressure exerted by pure water at 4 °C at a depth of 40 cm under the surface. The density of water at 4 °C is 1 000 kg·m^{-3}.

Answer
p = ρgh = (1 000)(9,8)(0,4) = 3 920 Pa

Activity 6.2: Fluid pressure

1. The property of a fluid that describes its internal resistance is known as:

   A Viscosity B Friction
   C Resistance D Internal energy

2. The pressure of the water at the bottom of a pond is … the pressure of the water at the surface.

   A higher than B lower than
   C equal to D either higher or lower than

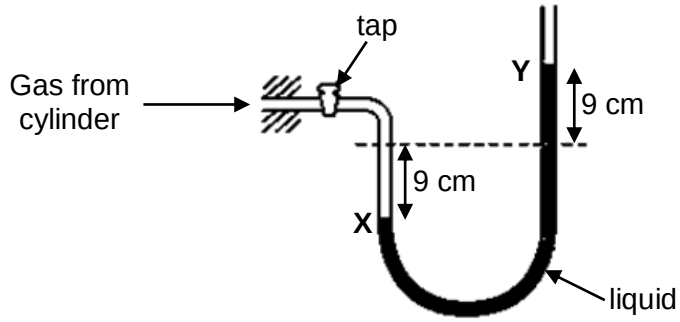
3. As we go to a higher altitude the atmospheric pressure …

   A decreases. B remains the same.
   C increases. D increases and then decreases.

4. Which of the following will cause the height of the mercury column in a barometer, **A**, to be higher than that of another barometer, **B**, located at sea level?

   A Bring **A** up a mountain.
   B Bring **A** to the moon and place it out in the open.
   C Bring **A** into a deep underground mine.
   D Introduce some air into barometer **A**, in the space above the mercury column.

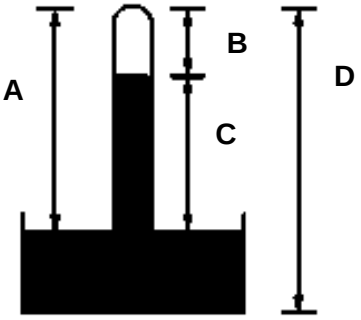
5. The diagram shows levels **X** and **Y** in a liquid manometer when the gas tap is opened.



   What is the pressure of the gas in the cylinder?

   A 9 cm of liquid above atmospheric pressure
   B 18 cm of liquid above atmospheric pressure
   C 9 cm of liquid below atmospheric pressure
   D 18 cm of liquid below atmospheric pressure

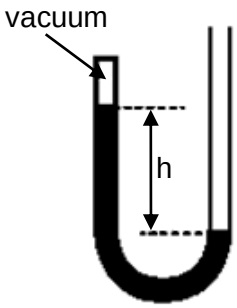
6. The diagram shows a simple mercury barometer.



Which height is a measure of atmospheric pressure?

A A B B C C D D

7. The diagram shows a manometer containing mercury that is sealed at one end.



What happens to the distance h when the manometer is taken to the top of a mountain?

A It increases, because atmospheric pressure increases with height.
B It decreases, because atmospheric pressure increases with height.
C It decreases, because atmospheric pressure decreases with height.
D It increases, because atmospheric pressure decreases with height.

8. An air bubble is inside an air-tight cylinder fitted with a piston. What happens to the air bubble when the piston is pushed in? Note that the air bubble is surrounded by air in the cylinder. Assume that there is still enough space for the air bubble to "survive" after the piston is pushed in.

A Explodes B Expands
C Vibrates D Contracts

9. If the pressure exerted by a liquid is 4 980 Pa at a depth of 2,4 m, calculate the density of the liquid. *(Answer: 211,73 kg·m^{-3})*

10. A swimming pool is 2,2 m deep at the deepest end. Calculate how far above the bottom of the pool a toy is if the pressure exerted by the water at that point is 18 130 Pa. *(Answer: 0,35 m)*

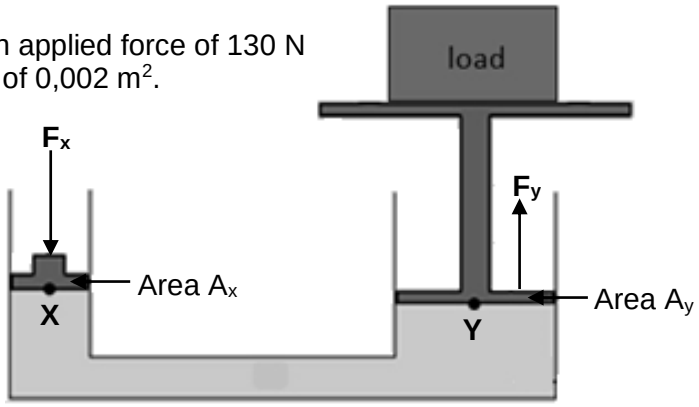
11. Calculate the pressure exerted by 200 mm of mercury.
The density of mercury is 13 593 kg·m^{-3}. *(Answer: 26 642,28 Pa)*

12. What is the total mass of the Earth's atmosphere? (The Earth is spherical shaped with the radius of the 6,37 x 10^6 m, and atmospheric pressure at the surface is 1,013 x 10^5 Pa).
Hint: the area of a sphere is $4\pi r^2$.

6.3 Pascal's law

Example 4

For the hydraulic system in the diagram, an applied force of 130 N is exerted on piston **X**, with a surface area of 0,002 m^2.



Calculate the:

(a) Pressure in the liquid

(b) Thrust on piston **Y** if the area of piston **B** is 0,025 m^2

Answer

(a) $p = \frac{F}{A} = \frac{130}{0,002} = 65\ 000$ Pa

(b) $F = pA = 65\ 000 \times 0,025 = 1\ 625$ N

Example 5

A 500 N weight sits on the small piston of a hydraulic machine. The small piston has area 2 cm^2. If the large piston has area 40 cm^2, how much weight can the large piston support?

Answer

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\therefore \frac{500}{2\times10^{-4}} = \frac{F_2}{4\times10^{-3}}$$

$\therefore F_2 = 10\ 000$ N

Activity 6.3: Pascal's law

1. A machine which works on Pascal's law is known as:

   A Vernier calliper B Hydraulic press
   C Barometer D Screw gauge

2. Which one of the following is not an application of Pascal's law?

   A Car brakes B Car lift
   C Electric fan D Blood pumped by the heart

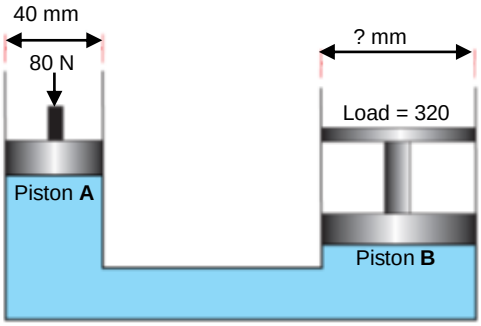
3. A hydraulic press allow large masses to be lifted with small forces as a result of which principle?

   A Pascal's B Bernoulli's
   C Huygens' D Archimedes

4. Which one of the following statements describes Pascal's law?
   A The area is inversely proportional to the pressure in it if the temperature remains constant.
   B The pressure exerted on the surface of the liquid in closed hydraulic system is transmitted equally in all directions.
   C The pressure is proportional to the volume if the temperature remains constant.
   D Volume is inversely proportional to the pressure on it if the temperature increases.

5. A hydraulic system is used to lift a 2 000 kg vehicle in a service station. If the vehicle sits on a piston of area 0,5 m^2, and a force is applied to a piston of area 0,03 m^2, what is the minimum force that must be applied to lift the vehicle. *(Answer: 1 176 N)*

6. A hydraulic system is used to compress metals for recycling. The specifications of the system are as shown in the diagram below.



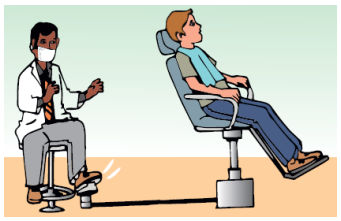
Calculate the:

6.1 Fluid pressure in the hydraulic system when in equilibrium *(Answer: 6,37 x 10^4 Pa)*

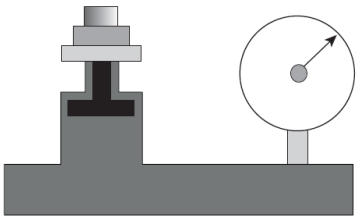
6.2 Diameter of piston B *(Answer: d = 80 mm)*

7. A barber raises his customer's chair by applying a force of 100 N to a hydraulic piston of area 0,01 m^2. If the chair is attached to a piston of area 0,1 m^2, how massive a customer can the chair raise? Assume the chair itself has a mass of 5 kg. *(Answer: 1 000 N)*

8. The figure below shows a dentist treating a patient with a mass of 54 kg. The dentist presses on the input piston to raise the patient. The cross-sectional areas of the input piston and the output piston are 400 cm^2 and 600 cm^2 respectively. How much force is exerted by the dentist in order to raise the patient? *(Answer: 352,8 N)*



9. A dead-weight tester is a device commonly used for calibrating pressure gauges. Weights loaded onto the piston carrier generate a known pressure in the piston cylinder, which in turn is applied to the gauge. The tester generates a pressure of 35 MPa when loaded with a 100 kg weight.



Calculate the:

9.1 Diameter of the piston cylinder *(Answer: 5,98 mm)*

9.2 Calculate the load (in kg) necessary to produce a pressure of 150 kPa *(Answer: 0,43 kg)*