MEMO SEQUENCES & SERIES

MAY/JUNE 2023

QUESTION 2/*VRAAG* 2

2.1.1	$\frac{1}{5}+\frac{1}{15}+\frac{1}{45}+\ldots$ $r=\frac{\frac{1}{15}}{\frac{1}{5}}=\frac{1}{3}$ $-1<\frac{1}{3}<1$ $\therefore$ the series is convergent.	✓ $r=\frac{1}{3}$ ✓ answer (any indicator of convergence) (2)
2.1.2	$S_\infty=\frac{a}{1-r}$ $=\frac{\frac{1}{5}}{1-\frac{1}{3}}$ $=\frac{3}{10}$	✓ substitution ✓ answer (2)
2.2.1	$4x\ ;\ \frac{1}{81}$	✓ $4x$ ✓ $\frac{1}{81}$ (2)
2.2.2	$T_n=x+(n-1)x$ $=x+xn-x$ $=xn$ Answer only: Full Marks	✓ substitution ✓ answer (2)
2.2.3	$T_n=ar^{n-1}$ $T_{13}=\frac{1}{3}\left(\frac{1}{3}\right)^{13-1}$ $T_{13}=\left(\frac{1}{3}\right)^{13}$ or $\frac{1}{1\,594\,323}$ or $6{,}27\times10^{-7}$ or 3^{-13}	✓ $n=13$ ✓ $r=\frac{1}{3}$ ✓ answer (3)
2.2.4	$\sum_{n=1}^{21}P_n=S_{11}+S_{10}$ $=\frac{11}{2}[2x+10x]+\frac{\frac{1}{3}\left[1-\left(\frac{1}{3}\right)^{10}\right]}{1-\frac{1}{3}}$ $=66x+0{,}5$ $33{,}5=66x+0{,}5$ $\therefore x=\frac{1}{2}$	✓ S_{11} ✓ $+S_{10}$ ✓ arithmetic sum ✓ geometric sum ✓ $66x+0{,}5$ (A) ✓ answer (6)
		[17]

QUESTION 3/*VRAAG* 3

3.1	$x\ ;\ x\ ;\ T_3\ ;\ \ldots$ $0 \qquad T_3-x$ 10 $2a=10 \qquad 3a+b=0$ $a=5 \qquad b=-15$ $T_3-x-0=10$ $\therefore T_3=x+10$ $2x+T_3=28$ $2x+x+10=28$ $3x=18$ $x=6$ $a+b+c=6$ $5-15+c=6$ $c=16$ $\therefore T_n=5n^2-15n+16$	✓ $2a=10$ ✓ $3a+b=0$ ✓ $T_3=x+10$ ✓ $2x+T_3=28$ ✓ $x=6$ ✓ $5-15+c=6$ (6)
	OR/*OF* $2a=10$ $\therefore a=5$ $T_1=a+b+c \qquad T_2=4a+2b+c \qquad T_3=9a+3b+c$ $=5+b+c \qquad =20+2b+c \qquad =45+3b+c$ $5+b+c=20+2b+c$ $b=-15$ $T_1=-10+c \qquad T_2=-10+c \qquad T_3=c$ $T_1+T_2+T_3=-10+c-10+c+c$ $28=3c-20$ $c=16$	**OR/*OF*** ✓ $2a=10$ ✓ $5+b+c=20+2b+c$ ✓ $T_1=-10+c$ ✓ $T_2=-10+c$ ✓ $28=3c-20$ ✓ $c=16$ (6)

NOV 2022

QUESTION 3/*VRAAG* 3

3.1	$x \;;\; x \;;\; T_3 \;;\; \ldots$ first differences: $0 \quad T_3 - x$ second difference: 10 $2a = 10 \qquad 3a + b = 0$ $a = 5 \qquad b = -15$ $T_3 - x - 0 = 10$ $\therefore T_3 = x + 10$ $2x + T_3 = 28$ $2x + x + 10 = 28$ $3x = 18$ $x = 6$ $a + b + c = 6$ $5 - 15 + c = 6$ $c = 16$ $\therefore T_n = 5n^2 - 15n + 16$	✓ $2a = 10$ ✓ $3a + b = 0$ ✓ $T_3 = x + 10$ ✓ $2x + T_3 = 28$ ✓ $x = 6$ ✓ $5 - 15 + c = 6$ (6)
	OR/*OF*	**OR/*OF***
	$2a = 10$ $\therefore a = 5$ $T_1 = a + b + c \qquad T_2 = 4a + 2b + c \qquad T_3 = 9a + 3b + c$ $= 5 + b + c \qquad = 20 + 2b + c \qquad = 45 + 3b + c$ $5 + b + c = 20 + 2b + c$ $b = -15$ $T_1 = -10 + c \qquad T_2 = -10 + c \qquad T_3 = c$ $T_1 + T_2 + T_3 = -10 + c - 10 + c + c$ $28 = 3c - 20$ $c = 16$	✓ $2a = 10$ ✓ $5 + b + c = 20 + 2b + c$ ✓ $T_1 = -10 + c$ ✓ $T_2 = -10 + c$ ✓ $28 = 3c - 20$ ✓ $c = 16$ (6)

NOV 2022

2.1.1	$a = 14$ $T_6 = 14r^5 = 448$ $r^5 = 32$ $\therefore r = 2$ Answer only: full marks	✓ $T_6 = 14r^5 = 448$ ✓ $r = 2$ (2)
2.1.2	$T_n = 14(2)^{n-1}$ $S_n = \frac{14(2^6 - 1)}{2 - 1}$ $S_6 = 882$ $114\,674 - 882 = 113\,792$ $113\,792 = 896(2^n - 1)$ $128 = 2^n$ $n = 7$	✓ substitution into correct formula ✓ $S_6 = 882$ ✓ $128 = 2^n$ ✓ 7 (4)
	OR/OF $S_n = \frac{a(r^n - 1)}{r - 1}$ $114\,674 = \frac{14(2^n - 1)}{2 - 1}$ $8\,191 = 2^n - 1$ $2^n = 8\,192$ $n = \log_2 8\,192$ $n = 13$ $\therefore$ 7 more terms must be added to the first 6 terms.	**OR/OF** ✓ substitution into correct formula ✓ $2^n = 8\,192$ ✓ $n = 13$ ✓ 7 (4)
2.1.3	$r = \frac{1}{2}$ OR $448r^5 = 14$ $\therefore r = \frac{1}{2}$ $S_\infty = \frac{a}{1 - r}$ $S_\infty = \frac{448}{1 - \frac{1}{2}}$ $S_\infty = 896$	✓ $r = \frac{1}{2}$ ✓ substitution ✓ answer (3)

2.2	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$ $T_1=\frac{1}{6}$ $\quad T_2=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}$ $d=\frac{3}{6}-\frac{1}{6}=\frac{1}{3}$ $\frac{121}{6}=\frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$ $\frac{121}{3}=n\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$ $\frac{121}{3}=\frac{1}{3}n^2$ $121=n^2$ $n=11$ $\therefore k=10$	✓ $T_1=\frac{1}{6}$ ✓ d ✓ substitution ✓ value of n ✓ value of k (5)
	OR/*OF*	**OR/*OF***
	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$ $a=\frac{1}{6}$ $l=\frac{1}{3}k+\frac{1}{6}$ $n=k+1$ $S_n=\frac{n}{2}[a+l]$ $\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{6}+\frac{1}{3}k+\frac{1}{6}\right]$ $\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{3}k+\frac{1}{3}\right]$ $\frac{121}{6}=\frac{k+1}{2}\left[\frac{k+1}{3}\right]$ $\frac{121}{6}=\frac{(k+1)^2}{6}$ $k+1=\pm\sqrt{121}$ $k+1=11$ $k=10$	✓ $a=\frac{1}{6}$ ✓ l ✓ $n=k+1$ ✓ $\frac{121}{6}=\frac{(k+1)^2}{6}$ ✓ value of k (5)
		[14]

NOV 2022

QUESTION 3/*VRAAG* 3

3.1	$3a+b=7$ $3+b=7$ $b=4$	✓ $3a+b=7$ ✓ $3+b=7$ (2)
	OR/OF $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$	**OR/OF** ✓ $T_2-T_1=7$ ✓ substitution (2)
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$ Answer only: full marks	✓ substitution ✓ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ Answer only: full marks	✓ first difference ✓ 2 ✓ $2p+5$ (3)
	OR/*OF* First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$	**OR/*OF*** ✓ first difference ✓ 2 ✓ $2p+5$ (3)
3.4	$157=2p+5$ $p=76$ $\therefore$ Between T_{76} and T_{77}	✓ $157=2p+5$ ✓ $p=76$ ✓ T_{76} and T_{77} (3)
	OR/OF $T_{n+1}-T_n=157$ $(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$ $n^2+2n+1+4n+4+9-n^2-4n-9=157$ $2n=152$ $n=76$ $\therefore$ Between T_{76} and T_{77}	**OR/OF** ✓ $T_{n+1}-T_n=157$ ✓ $n=76$ ✓ T_{76} and T_{77} (3)
		[10]

Arithmetic series and sequences

June 2018

Q2.1.1

37 ; 50

Q2.1.2

$$a = \frac{\text{second difference}}{2} = \frac{2}{2} = 1$$

$$3a + b = 5$$
$$3 + b = 5$$
$$b = 2$$
$$a + b + c = 5$$
$$1 + 2 + c = 5$$
$$c = 2$$
$$T_n = an^2 + bn + c$$
$$= n^2 + 2n + 2$$

Q2.1.3

$$n^2 + 2n + 2 = 1765$$
$$n^2 + 2n - 1763 = 0$$
$$(n + 43)(n - 41) = 0$$
$$n = -43 \text{ or } n = 41$$

N/A

Q2.2

Sum of all multiples of 7 from 35 to 196:

$$a = 35; \quad d = 7$$
$$S_n = \frac{n}{2}[a + \ell]$$
$$= \frac{24}{2}[35 + 196]$$
$$= 12[231]$$
$$= 2772$$

Sum of all the natural numbers from 35 to 196:

$$a = 35; \quad d = 1; \quad n = 162$$
$$S_n = \frac{n}{2}[a + \ell]$$
$$= \frac{162}{2}[35 + 196]$$
$$= 81[231]$$
$$= 18\ 711$$

Sum of numbers not divisible by 7/
Som van getalle nie deelbaar deur 7

$$= 18\ 711 - 2772$$
$$= 15\ 939$$

March 2018

Q3.1

$-1 ; \ 2 ; 5$

$$T_n = -1 + (n - 1)(3)$$
$$= 3n - 4$$

Q3.2

$$T_{43} = 3(43) - 4$$
$$= 125$$

Q3.3

$$T_n = 3n - 4$$
$$\sum_{k=1}^{n} T_k = 3(1) - 4 + 3(2) - 4 + 3(3) - 4 + \dots\dots\dots + 3n - 4$$
$$= 3(1 + 2 + 3 + \dots\dots + n) - 4n$$
$$= \frac{3n(n + 1)}{2} - 4n$$
$$= \frac{3n^2 - 5n}{2}$$

Q3.4

$$T_{11} = (T_{11} - T_{10}) + (T_{10} - T_9) + (T_9 - T_8) + \dots + (T_3 - T_2) + (T_2 - T_1) + T_1$$

$$125 = 29 + 26 + 23 + \dots 2 + T_1$$
$$= \frac{10}{2}(29 + 2) + T_1$$
$$= 155 + T_1$$
$$T_1 = -30$$

November 2017:

Q2.1.1

first differences: –9; –15; –21
second difference = – 6

Q2.1.2

$T_n = an^2 + bn + c$

$a = \frac{\text{second difference}}{2} = -3$

$3a + b = -9$

$3(-3) + b = -9$

$b = 0$

$a + b + c = 5$

$-3 + 0 + c = 5$

$c = 8$

$T_n = -3n^2 + 8$

Q2.1.3

$-3n^2 + 8 = -25\,939$

$-3n^2 = -25947$

$n^2 = 8649$

$n = -93 \quad or \quad n = 93$

The 93rd term has a value of –25 939

June 2017:

Q3.1

First differences: 17; 15
Second difference: –2

$T_n = an^2 + bn + c$

$a = \frac{\text{second difference}}{2} = \frac{-2}{2} = -$

$3a + b = 17$

$3(-1) + b = 17$

$b = 20$

$a + b + c = 0$

$-1 + 20 + c = 0$

$c = -19$

$T_n = -n^2 + 20n - 19$

Q3.2

$56 = -n^2 + 20n - 19$

$n^2 - 20n + 75 = 0$

$(n - 15)(n - 5) = 0$

$n = 5 \text{ or } n = 15$

Q3.3

$\sum_{n=5}^{10} T_n - \sum_{n=11}^{15} T_n$

$= T_5 + T_6 + T_7 + T_8 + T_9 + T_{10} - T_{11} - T_{12} - T_{13} - T_{14} - T$

$= (T_5 - T_{15}) + (T_6 - T_{14}) + \ldots + (T_9 - T_{11}) + T_{10}$

$= T_{10}$

because by symmetry $T_5 = T_{15}$; $T_6 = T_{14}$ ...

$T_{10} = -(10)^2 + 20(10) - 19$

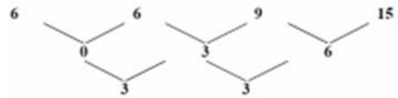
$= 81$

March 2017:

Q3.1.1

24

Q3.1.2



$2a = 3 \qquad 3a + b = 0$

$a = \frac{3}{2} \qquad b = -\frac{9}{2}$

$T_n = \frac{3}{2}n^2 - \frac{9}{2}n + 9$

$a + b + c = 6$

$c = 9$

Q3.1.3

$\frac{3}{2}n^2 - \frac{9}{2}n + 9 = 3249$

$3n^2 - 9n + 18 = 6498$

$3n^2 - 9n - 6480 = 0$

$n^2 - 3n - 2160 = 0$

$(n + 45)(n - 48) = 0$

$n \neq -45 \quad or \quad n = 48$

Q3.2

$-1\ ;\ 2\sin 3x\ ;\ 5;\ldots$

$2\sin 3x + 1 = 5 - 2\sin 3x$

$4\sin 3x = 4$

$\sin 3x = 1$

$3x = 90^\circ$

$x = 30^\circ$

Nov 2019

Q3.1

$$\sum_{y=3}^{10}\frac{1}{y-2}-\sum_{y=3}^{10}\frac{1}{y-1}$$
$$=\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}\right)-\left(\frac{1}{2}+\frac{1}{3}+....+\frac{1}{8}+\frac{1}{9}\right)$$
$$=1-\frac{1}{9}$$
$$=\frac{8}{9}$$

Nov 2019

Q2.2.1

$$a=\frac{5}{8}\quad;\quad r=\frac{1}{2}\quad;\quad n=21$$
$$S_n=\frac{a(1-r^n)}{1-r}$$
$$S_{21}=\frac{\frac{5}{8}\left(1-\left(\frac{1}{2}\right)^{21}\right)}{1-\frac{1}{2}}$$
$$=1{,}2499...$$
$$=1{,}25$$

Q2.2.2

$$T_n>\frac{5}{8192}$$
$$ar^{n-1}>\frac{5}{8192}$$
$$\frac{5}{8}\left(\frac{1}{2}\right)^{n-1}>\frac{5}{8192}$$
$$\left(\frac{1}{2}\right)^{n-1}>\frac{1}{1024}$$
$$\left(\frac{1}{2}\right)^{n-1}>\left(\frac{1}{2}\right)^{10}$$
$$\therefore n-1<10$$
$$n<11$$
$$\therefore n=10$$

May-June 2019

Q2.2.1

$$r=\frac{-18}{36}=-\frac{1}{2}$$

Q2.2.2

$$T_n=36\left(-\frac{1}{2}\right)^{n-1}$$
$$\frac{9}{4096}=36\left(-\frac{1}{2}\right)^{n-1}$$
$$\frac{1}{16384}=\left(-\frac{1}{2}\right)^{n-1}$$
$$\left(-\frac{1}{2}\right)^{14}=\left(-\frac{1}{2}\right)^{n-1}$$
$$14=n-1$$
$$n=15$$

Q2.2.3

$$S_\infty=\frac{a}{1-r}$$
$$=\frac{36}{1-\left(-\frac{1}{2}\right)}$$
$$=24$$

Q2.2.4

$$\frac{T_1+T_3+T_5+T_7+...+T_{499}}{T_2+T_4+T_6+T_8+...+T_{500}}$$
$$=\frac{a+ar^2+ar^4+...+ar^{498}}{ar+ar^3+ar^5+...+ar^{499}}$$
$$=\frac{a+ar^2+ar^4+...+ar^{498}}{r(a+ar^2+ar^4+...+ar^{498})}$$
$$=\frac{1}{r}$$

Nov 2018

Q3.1

$r=\frac{1}{2}$ and $S_\infty=6$

$S_\infty=\frac{a}{1-r}$

$6=\frac{a}{1-\frac{1}{2}}$

$a=3$

Q3.2

$T_n=ar^{n-1}$

$T_8=3\left(\frac{1}{2}\right)^7$

$T_8=\frac{3}{128}$

Q3.3

$\sum_{k=1}^{n}3(2)^{1-k}=5{,}8125$

$3+\frac{3}{2}+\frac{3}{4}+\ldots=5{,}8125$

$S_n=\frac{a(1-r^n)}{1-r}=5{,}8125$

$\frac{3\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}}=5{,}8125$

$6\left[1-\left(\frac{1}{2}\right)^n\right]=5{,}8125$

$\left(\frac{1}{2}\right)^n=\frac{1}{32}=0{,}03125$

$2^{-n}=2^{-5}$

$n=5$

Q3.4

$\sum_{k=1}^{20}3(2)^{1-k}=p$

$\sum_{k=1}^{20}6(2)^{-k}=p$

$\therefore\sum_{k=1}^{20}24(2)^{-k}=4p$

June 2018

Q3.1

$r=0{,}94;\quad a=100$

$T_3=ar^2$

$=100(0{,}94)^2$

$=88{,}36\text{ km}$

Q3.2

$S_n=\frac{a(r^n-1)}{r-1}$

$750=\frac{100(0{,}94^n-1)}{0{,}94-1}$

$\frac{750(-0{,}06)}{100}=0{,}94^n-1$

$0{,}94^n=1-\frac{9}{20}$

$0{,}94^n=0{,}55$

$n=\frac{\log 0{,}55}{\log 0{,}94}$

$=9{,}66$

He will pass halfway mark on the tenth day.

Q3.3

$S_\infty=\frac{a}{1-r}$

$1500<\frac{100}{1-r}$

$1-r<\frac{100}{1500}$

$r>\frac{14}{15}$ or $93{,}33\%$

March 2018

Q2.1.1

$30\,;\,10\,;\,\frac{10}{3}\ldots\ldots$

$a=30\qquad r=\frac{1}{3}$

$T_n=ar^{n-1}$

$\frac{10}{729}=30\left(\frac{1}{3}\right)^{n-1}$

$\frac{1}{2187}=3^{1-n}$

$3^{-7}=3^{1-n}$

$-7=1-n$

$n=8$

Q2.1.2

$$S_\infty = \frac{a}{1-r}$$
$$= \frac{30}{1-\frac{1}{3}}$$
$$= 45$$

Q2.2

$$S_n = a + (a+d) + \ldots\ldots + (a + (n-2)d) + T_n$$
$$S_n = T_n + (T_n - d) + (T_n - 2d) + \ldots\ldots\ldots + a$$
$$2S_n = (a+T_n) + (a+T_n) + (a+T_n) + \ldots\ldots + (a+T_n)$$
$$S_n = \frac{n}{2}(a+T_n)$$
but $Tn = a + (n-1)d$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Q2.2

$$S_n = a + (a+d) + \ldots\ldots + (a + (n-2)d) + T_n$$
$$S_n = T_n + (T_n - d) + (T_n - 2d) + \ldots\ldots\ldots + a$$
$$2S_n = (a+T_n) + (a+T_n) + (a+T_n) + \ldots\ldots + (a+T_n)$$
$$S_n = \frac{n}{2}(a+T_n)$$
but $Tn = a + (n-1)d$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

November 2017:

Q3.1

$$a + ar = 2$$
$$a(1+r) = 2$$
$$a = \frac{2}{1+r}$$

Q3.2

$$S_\infty = T_1 + T_2 + \sum_{n=3}^{\infty} T_n$$
$$S_\infty = 2 + \frac{1}{4}$$
$$\frac{a}{1-r} = 2 + \frac{1}{4}$$
$$\frac{a}{1-r} = \frac{9}{4}$$
$$\left(\frac{2}{1+r}\right) \times \left(\frac{1}{1-r}\right) = \frac{9}{4}$$
$$\frac{2}{1-r^2} = \frac{9}{4}$$
$$8 = 9 - 9r^2$$
$$9r^2 = 1$$
$$r = \frac{1}{3}$$
$$a = \frac{3}{2}$$

June 2017:

Q2.1.1

$3;\ 2;\ k;\ \ldots$

$r = \frac{2}{3}$

Q2.1.2

$$r = \frac{T_3}{T_2}$$

$$T_3 = r \times T_2$$

$$= \frac{2}{3} \times 2$$

$$= \frac{4}{3}$$

Thus $k = \frac{4}{3}$

Q2.1.3

$$T_n = a.r^{n-1}$$

$$\frac{128}{729} = 3 \times \left(\frac{2}{3}\right)^{n-1}$$

$$\left(\frac{2}{3}\right)^{n-1} = \frac{128}{2187}$$

$$\left(\frac{2}{3}\right)^{n-1} = \left(\frac{2}{3}\right)^{7}$$

$$n - 1 = 7$$

$$n = 8$$

March 2017:

Q2.1

For geometric:

$$-\frac{1}{4};\ b;\ -1;\ \ldots\ldots$$

$$\frac{b}{-\frac{1}{4}} = -\frac{1}{b}$$

$$b^2 = \frac{1}{4}$$

$$b = \pm\frac{1}{2}$$

Q2.2

$$-\frac{1}{4};\ \frac{1}{2};\ -1;\ \ldots\ldots$$

$$r = -2$$

$$T_{19} = ar^{18}$$

$$= \left(-\frac{1}{4}\right)(-2)^{18}$$

$$= \left(-\frac{2^{18}}{2^2}\right)$$

$$= -2^{16}$$

$$= -65536$$

Q2.3

The series is:

$$-\frac{1}{4};\ \frac{1}{2};\ -1;\ 2;\ -4;\ 8;\ \ldots$$

The new positive series is:

$$\frac{1}{2};\ 2;\ 8;\ 32;\ 128$$

$$a = \frac{1}{2} \qquad r = 4$$

$$\sum_{n=1}^{20}\left(\frac{1}{2}\right)(4)^{n-1}$$

Or you could write it as:

$$\sum_{p=0}^{19}\left(\frac{1}{2}\right)(4)^{p}$$

Q2.4

No, the series is not convergent

$r = 4$ and for convergence $-1 < r < 1$

2.2	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$	
	$T_1=\frac{1}{6}$ $\quad T_2=\frac{1}{3}+\frac{1}{6}=\frac{3}{6}$	✓ $T_1=\frac{1}{6}$
	$d=\frac{3}{6}-\frac{1}{6}=\frac{1}{3}$	✓ d
	$\frac{121}{6}=\frac{n}{2}\left[2\left(\frac{1}{6}\right)+(n-1)\left(\frac{1}{3}\right)\right]$	✓ substitution
	$\frac{121}{3}=n\left[\frac{1}{3}+\frac{1}{3}n-\frac{1}{3}\right]$	
	$\frac{121}{3}=\frac{1}{3}n^2$ $121=n^2$	
	$n=11$ $\therefore k=10$	✓ value of n ✓ value of k (5)
	OR/*OF*	**OR/*OF***
	$\sum_{p=0}^{k}\left(\frac{1}{3}p+\frac{1}{6}\right)=20\frac{1}{6}$	
	$a=\frac{1}{6}$	✓ $a=\frac{1}{6}$
	$l=\frac{1}{3}k+\frac{1}{6}$	✓ l
	$n=k+1$	✓ $n=k+1$
	$S_n=\frac{n}{2}[a+l]$	
	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{6}+\frac{1}{3}k+\frac{1}{6}\right]$	
	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{1}{3}k+\frac{1}{3}\right]$	
	$\frac{121}{6}=\frac{k+1}{2}\left[\frac{k+1}{3}\right]$	
	$\frac{121}{6}=\frac{(k+1)^2}{6}$	✓ $\frac{121}{6}=\frac{(k+1)^2}{6}$
	$k+1=\pm\sqrt{121}$ $k+1=11$ $k=10$	✓ value of k (5)
		[14]

NOV 2022

QUESTION 3/*VRAAG* 3

3.1	$3a+b=7$ $3+b=7$ $b=4$	✓ $3a+b=7$ ✓ $3+b=7$ (2)
	OR/OF $T_2-T_1=7$ $4+2b+9-(1+b+9)=7$ $b=4$	**OR/OF** ✓ $T_2-T_1=7$ ✓ substitution (2)
3.2	$T_n=n^2+4n+9$ $T_{60}=(60)^2+4(60)+9$ $=3849$ Answer only: full marks	✓ substitution ✓ answer (2)
3.3	14 ; 21 ; 30 ; 41; First difference: 7 ; 9 ; 11 ; … Common 2nd difference: 2 $T_p=2p+5$ Answer only: full marks	✓first difference ✓ 2 ✓ $2p+5$ (3)
	OR/*OF* First difference: 7 ; 9 ; 11 ; … $T_n=a+(n-1)d$ $T_p=7+(p-1)(2)$ $T_p=2p+5$	**OR/*OF*** ✓first difference ✓ 2 ✓ $2p+5$ (3)
3.4	$157=2p+5$ $p=76$ $\therefore$ Between T_{76} and T_{77}	✓ $157=2p+5$ ✓ $p=76$ ✓ T_{76} and T_{77} (3)
	OR/OF $T_{n+1}-T_n=157$ $(n+1)^2+4(n+1)+9-(n^2+4n+9)=157$ $n^2+2n+1+4n+4+9-n^2-4n-9=157$ $2n=152$ $n=76$ $\therefore$ Between T_{76} and T_{77}	**OR/OF** ✓ $T_{n+1}-T_n=157$ ✓ $n=76$ ✓ T_{76} and T_{77} (3)
		[10]