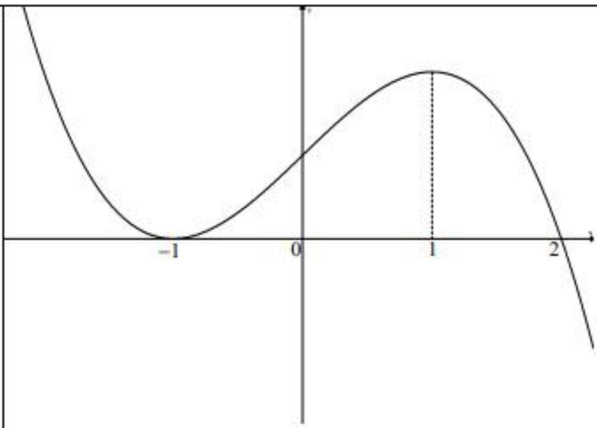
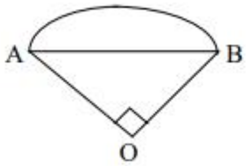
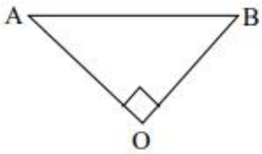
MEMORANDUM APPLICATION OF CALCULUS

May – June 2021

QUESTION/*VRAAG* 10

10.1		✓ $x=-1$ and $x=2$ ✓ TP at $x=-1$ ✓ TP at $x=1$ ✓ shape (4)
10.2.1	 Area of segment = $\frac{1}{4}$ Area of big circle $=\frac{1}{4}\pi\ (x-x^2)^2$  Area triangle ABO counted = Area $\Delta=\frac{1}{2}(x-x^2)^2$ Area of shaded region $=\frac{1}{4}\pi(x-x^2)^2-\frac{1}{2}(x-x^2)^2$ $=\frac{\pi-2}{4}(x-x^2)^2$ $=\left(\frac{\pi-2}{4}\right)(x^2-2x^3+x^4)$	✓✓ $\frac{1}{4}\pi(x-x^2)^2$ ✓ Area $\Delta=\frac{1}{2}(x-x^2)^2$ ✓ subtract areas ✓ common factor (5)

10.2.2	Area of shaded region $=\frac{(\pi-2)}{4}\left(x^4-2x^3+x^2\right)$ $\frac{dA}{dx}=\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$ $4x^3-6x^2+2x=0$ $x(2x^2-3x+1)=0$ $x(2x-1)(x-1)=0$ $x\neq 0$ or $x=\frac{1}{2}$ or $x\neq 1$	✓ $\left(\frac{\pi-2}{4}\right)\left(4x^3-6x^2+2x\right)$ ✓ factors ✓ $x=0;x=1;x=\frac{1}{2}$ ✓ $x=\frac{1}{2}$ (4)
		[13]

MAY – June 2017

QUESTION/*VRAAG* 10

10.1	$60=2b+2r+\frac{1}{2}(2\pi r)$ $2b=60-2r-\pi r$ $b=30-r-\frac{1}{2}\pi r$	✓ $60=2b+2r+\frac{1}{2}(2\pi r)$ ✓ $b=30-r-\frac{1}{2}\pi r$ (2)
10.2	Area = area of rectangle + area of semicircle $A(r)=\text{length}\times\text{breadth}+\frac{1}{2}(\text{area of circle})$ $=(2r)\left(30-r-\frac{1}{2}\pi r\right)+\frac{1}{2}\left(\pi r^2\right)$ $=60r-2r^2-\pi r^2+\frac{1}{2}\pi r^2$ $=60r-2r^2-\frac{1}{2}\pi r^2$ $=60r-\left(2+\frac{1}{2}\pi\right)r^2$ For a maximum, $A'(r)=0$ $60-2\left(2+\frac{1}{2}\pi\right)r=0$ $60-(4+\pi)r=0$ $r=\frac{60}{4+\pi}$ $=8{,}40$ m	✓ $(2r)\left(30-r-\frac{1}{2}\pi r\right)$ ✓ $\frac{1}{2}\left(\pi r^2\right)$ ✓ $60r-2r^2-\frac{1}{2}\pi r^2$ ✓ $A'(r)=0$ ✓ $60-2\left(2+\frac{1}{2}\pi\right)r$ ✓answer (6) **[8]**

QUESTION/*VRAAG* 10

10.1	$\frac{h}{r} = \tan 60^0$ $r = \frac{h}{\tan 60^0}$ $\therefore r = \frac{h}{\sqrt{3}}$	✓ $\frac{h}{r} = \tan 60^0$ ✓answer (2)
10.2	$V_{cone} = \frac{1}{3}\pi r^2 h$ $= \frac{1}{3}\pi\left(\frac{h}{\sqrt{3}}\right)^2 h$ $= \frac{1}{9}\pi h^3$ $\frac{dV}{dh} = \frac{1}{3}\pi h^2$ $\left.\frac{dV}{dh}\right\|_{h=9} = \frac{1}{3}\pi(9)^2$ $= 27\pi$ or $84{,}82\ cm^3/cm$	✓formula ✓substitution of the value of *r* in terms of *h* ✓simplified volume answer ✓derivative ✓answer (5) **[7]**

QUESTION 11

11.1	Length of box $= 3x$ Volume $= l \times b \times h$ $9 = 3x \cdot x \cdot h$ $9 = 3x^2 h$ $h = \frac{3}{x^2}$	✓ length of box $= 3x$ ✓ $9 = 3x \cdot x \cdot h$ ✓ $h = \frac{3}{x^2}$ (3)
11.2	$C = (2(3xh) + 2xh) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\frac{3}{x^2}\right) \times 50 + 600x^2$ $= \frac{1200}{x} + 600x^2$ **OR** $C = (h \times 8x) \times 50 + (2 \times 3x^2) \times 100$ $= 8x\left(\frac{3}{x^2}\right) \times 50 + 600x^2$ $= \frac{1200}{x} + 600x^2$	✓ $(2(3xh) + 2xh) \times 50$ ✓ $(2 \times 3x^2) \times 100$ ✓ substitution of $h = \frac{3}{x^2}$ (3) ✓ $(h \times 8x) \times 50$ ✓ $(2 \times 3x^2) \times 100$ ✓ substitution of $h = \frac{3}{x^2}$ (3)
11.3	$C = 1200x^{-1} + 600x^2$ $\frac{dC}{dx} = -1200x^{-2} + 1200x$ $0 = -1200x^{-2} + 1200x$ $1200x^3 = 1200$ $x^3 = 1$ $x = 1$ Therefore the width of the box is 1 metre.	✓ $\frac{dC}{dx} = -1200x^{-2} + 1200x$ ✓ $\frac{dC}{dx} = 0$ ✓ $x^3 = 1$ ✓ $x = 1$ (4) **[10]**

NOV 2019

Q8.1

36cm

Q8.2

$\therefore t = 6$

only once

Q8.3

$h(t) = -2t^3 + 15t^2 - 24t + 36$

$h'(t) = -6t^2 + 30t - 24$

$-6t^2 + 30t - 24 = 0$

$t^2 - 5t + 4 = 0$

$(t-4)(t-1) = 0$

$t = 4$ or $t = 1$

Only $t = 4$ because maximum value required

$h = -2(4)^3 + 15(4)^2 - 24(4) + 36 = 52\ cm$

Q9.1

$f'(x) = 9x^2$

$3x^3 = 9x^2$

$3x^3 - 9x^2 = 0$

$3x^2(x-3) = 0$

$x = 0$ or $x = 3$

Q9.2.1

For f and f'

Q9.2.2

The point (0 ; 0) is :

A point of inflection of f

A turning point of f'

Q9.3

$f''(x) = 18x$

$\text{Distance} = f''(1) - f'(1)$

$= 18(1) - 9(1)^2$

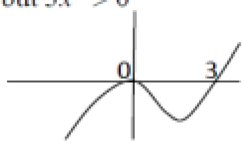
$= 9$

Q9.4

$3x^3 - 9x^2 < 0$

$3x^2(x-3) < 0$

but $3x^2 > 0$



$\therefore x - 3 < 0$

$\therefore x < 3\ ,\ x \neq 0$

QUESTION/*VRAAG* 9

9.1	$340 = \pi r^2 h$ $\therefore h = \dfrac{340}{\pi r^2}$	✓substitution into volume formula ✓answer (2)
9.2	$A = 2\pi r^2 + 2\pi rh$ $= 2\pi r^2 + 2\pi r\left(\dfrac{340}{\pi r^2}\right)$ $= 2\pi r^2 + 680r^{-1}$	✓formula ✓substitution of h (2)
9.3	$A(r) = 2\pi r^2 + 680r^{-1}$ $A'(r) = 4\pi r - 680r^{-2}$ $4\pi r - 680r^{-2} = 0$ $4\pi r = \dfrac{680}{r^2}$ $r^3 = \dfrac{680}{4\pi}$ $r = \sqrt[3]{\dfrac{680}{4\pi}}$ cm or 3,78 cm	✓ $4\pi r$ ✓ $-680r^{-2}$ ✓ $r^3 = \dfrac{680}{4\pi}$ ✓answer (4) **[8]**