# ANALYTICAL GEOMETRY NOTES

MODULE 9: Analytical Geometry

1. **Gradient:** $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

2. **Distance between two points:** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

3. **Coordinates of midpoint:** $\left(\dfrac{x_1 + x_2}{2} ; \dfrac{y_1 + y_2}{2} \right)$

4. **Equation of line:** $y = mx + c$ or $y - y_1 = m(x - x_1)$

5. **Inclination of line:** $\tan \theta = m$
   (angle between line and positive x-axis)

6. **Collinear points:** A, B, C : $m_{AB} = m_{BC}$

7. **Parallel lines:** $m_1 = m_2$

8. **Perpendicular lines:** $m_1 \times m_2 = -1$

9. **Median:** Line joining a vertex to the **midpoint** of the opposite side.
   (bisects the area of a Δ)

10. **Altitude**: Line from vertex, **perpendicular** to opposite side.

11. **Perpendicular bisector:** perpendicular line through midpoint.

12. **Parallelogram:** both pairs of opp. sides ||

13. **Rhombus:** parallelogram with one pair of adjacent sides =

14. **Rectangle:** parallelogram with one angle = 90°

15. **Trapezium:** quadrilateral with one pair of opp. sides ||

Determine the size of α
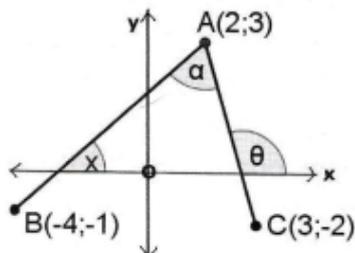
1. $m_{AC} = \dfrac{5}{-1}$ ∴ $\tan \theta = -5$
   ∴ $\theta = 180° - 78,69..°$
   $= 101,309...°$

2. $m_{AB} = \dfrac{4}{6}$ ∴ $\tan x = \dfrac{2}{3}$
   ∴ $x = 33,69....°$

3. ∴ $\alpha = 101,30..° - 33,68..°$
   $= 67,6°$ (ext. ∠ Δ)

4. <u>**Given, the equation of the circle.**</u>
   <u>**Find the centre and the radius of the circle.**</u>

   Given: $x^2 - 8x + y^2 - 10y - 59 = 0$

   1. Leave space for completing the square:

      $x^2 - 8x + \ldots\ldots + y^2 - 10y + \ldots\ldots = 59$

   2. **Complete the square**:

      $x^2 - 8x + \mathbf{16} + y^2 - 10y + \mathbf{25} = 59 + \mathbf{16} + \mathbf{25}$

   3. Write in **centre-radius equation** form:

      $(x - \mathbf{4})^2 + (y - \mathbf{5})^2 = 100$

      $\therefore$ centre $= (4;5)$
      radius $= 10$

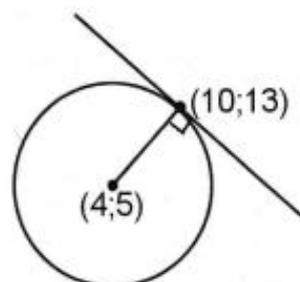**5.** <u>**Determine the equation of the tangent to a circle.**</u>

   $x^2 - 8x + y^2 - 10y - 59 = 0$ at the point $(10;13)$.

   1. Determine the centre of the circle as in example 4.

      $\therefore (x - 4)^2 + (y - 5)^2 = 100$
      $\therefore$ Centre $= (4;5)$
      Point $(10;13)$

   2. $m_{radius} = \dfrac{8}{6} = \dfrac{4}{3}$

      | **tangent $\perp$ radius** |
      | $m_1 \times m_2 = -1$ |

   3. $\therefore m_{tangent} = -\dfrac{3}{4}$

   4. $\therefore$ equation$_{tangent}$: $\quad y - 13 = -\dfrac{3}{4}(x - 10)$

      $y = -\dfrac{3}{4}x + 20\dfrac{1}{2}$