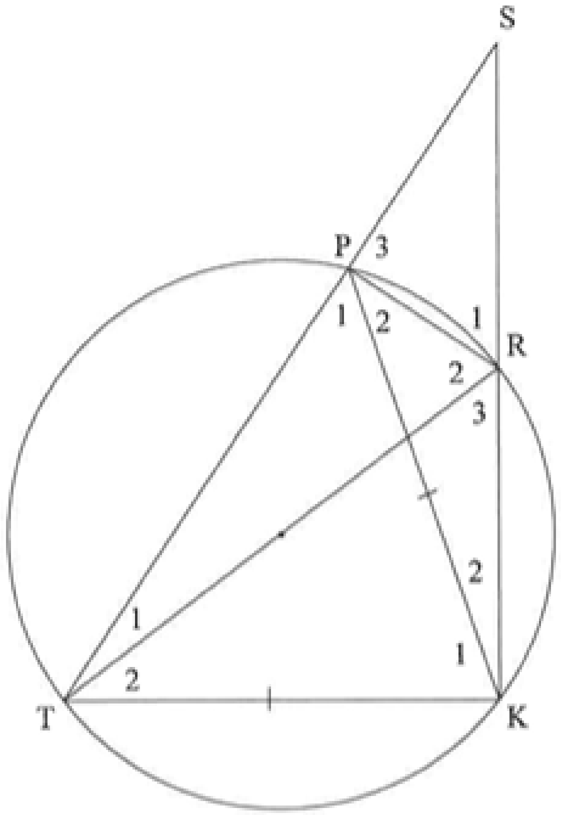
SIMILARITY QUESTIONS AND ANSWERS

MAY / JUNE 2023

QUESTION 10

In the diagram, TR is a diameter of the circle. PRKT is a cyclic quadrilateral. Chords TP and KR are produced to intersect at S. Chord PK is drawn such that PK = TK.



10.1 Prove, giving reasons, that:

10.1.1 SR is a diameter of a circle passing through points S, P and R (4)

10.1.2 $\hat{S} = \hat{P}_2$ (5)

10.1.3 $\Delta SPK \parallel\!\mid \Delta PRK$ (3)

10.2 If it is further given that SR = RK, prove that $ST = \sqrt{6}RK$. (5)

[17]

10.1.1	$T\hat{P}R = 90°$ [$\angle$ in semi-circle] $S\hat{P}R = 90°$ [$\angle$'s on a straight line] $\therefore$ SR is a diameter [converse $\angle$ in semi-circle] **OR** $T\hat{K}R = 90°$ [$\angle$ in semi-circle] $S\hat{P}R = 90°$ [ext $\angle$ of cyclic quad] $\therefore$ SR is a diameter [converse $\angle$ in semi-circle] **OR** [chord subtends a right angle]	✓S ✓R ✓S ✓R (4) ✓S ✓R ✓S ✓R (4)

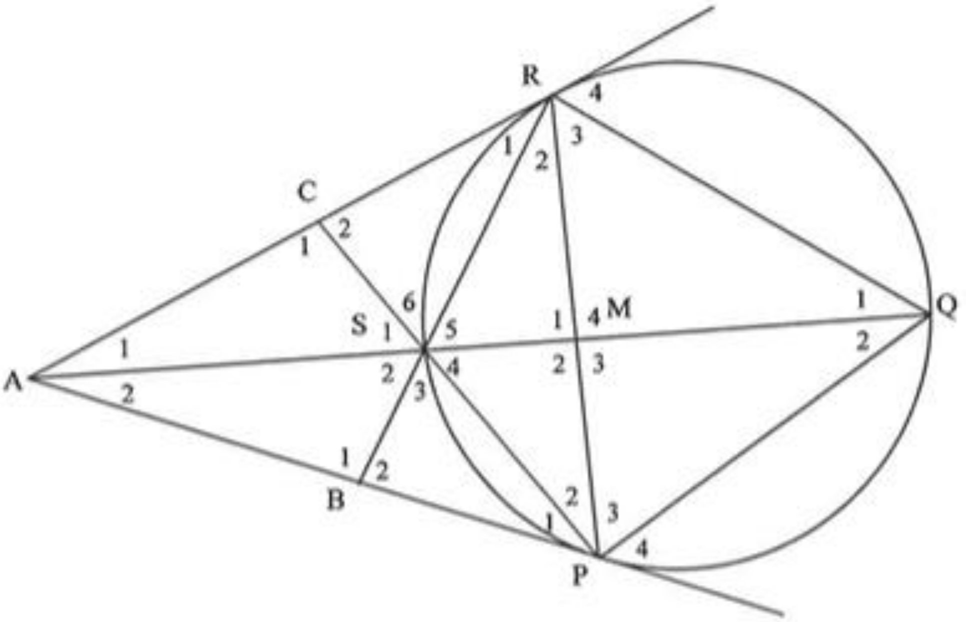
10.1.2	$\hat{R}_1 = P\hat{T}K$ [ext $\angle$ of cyclic quad] $\hat{P}_1 = P\hat{T}K = \hat{R}_1$ [$\angle$s opp equal sides] $\hat{S} + \hat{R}_1 = \hat{P}_1 + P_2$ [ext $\angle$ of Δ] $\therefore\ \hat{S} = \hat{P}_2$ [$\hat{R}_1 = \hat{P}_1$]	✓S ✓R ✓S /R ✓S ✓R (5)
10.1.3	In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $\Delta SPK \mid\mid\mid \Delta PRK$ [$\angle, \angle, \angle$] **OR**/***OF*** In ΔSPK and ΔPRK $\hat{S} = \hat{P}_2$ [proved] $\hat{K}_2 = \hat{K}_2$ [common] $S\hat{P}K = P\hat{R}K$ [sum of $\angle$s in Δ] $\Delta SPK \mid\mid\mid \Delta PRK$	✓S ✓S ✓S/R (3) ✓S ✓S ✓S/R (3)
10.2	$\frac{PK}{RK} = \frac{SK}{PK}$ [$\Delta SPK \mid\mid\mid \Delta PRK$] $PK^2 = SK.RK$ $ST^2 = SK^2 + TK^2$ [Pythagoras] TK = PK [Given] $ST^2 = SK^2 + PK^2$ $ST^2 = SK^2 + SK.RK$ $ST^2 = (2RK)^2 + 2RK.RK$ $ST^2 = 6RK^2$ $ST = \sqrt{6}RK$	✓S ✓S ✓ $PK^2 = SK.RK$ ✓SK = 2RK ✓ $ST^2 = 6RK^2$ (5)
		[17]

SIMILARITY QUESTIONS AND ANSWERS

NOVEMBER 2023

QUESTION 10

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.1 $\hat{S}_3 = \hat{S}_4$ (5)

10.2 SMRC is a cyclic quadrilateral (4)

10.3 RP is a tangent to the circle passing through P, S and A at P (6)

[15]

10.1	$\hat{S}_3 = P\hat{Q}R$	[ext ∠ of cyclic quad]	✓ S ✓ R
	$\hat{R}_3 = P\hat{Q}R$	[∠s opp equal sides]	✓ S / R
	$\therefore \hat{S}_3 = \hat{R}_3$		
	But $\hat{S}_4 = \hat{R}_3$	[∠s in the same seg]	✓ S ✓ R
	$\therefore \hat{S}_3 = \hat{S}_4$		(5)
10.2	$\hat{R}_1 + \hat{R}_2 = P\hat{Q}R$	[tan chord theorem]	✓ S ✓ R
	$\hat{S}_4 = P\hat{Q}R$	[proved in 10.1]	
	$\therefore \hat{S}_4 = \hat{R}_1 + \hat{R}_2$		✓ S
	SMRC is a cyclic quad	[converse ext ∠ of cyclic quad]	✓ R
			(4)
10.3	$\hat{S}_3 = \hat{R}_2 + \hat{P}_2$	[ext ∠ of Δ]	✓ S ✓ R
	$\hat{S}_4 = \hat{P}_1 + \hat{A}_2$	[ext ∠ of Δ]	✓ S
	$\therefore \hat{R}_2 + \hat{P}_2 = \hat{A}_2 + \hat{P}_1$		
	But $\hat{P}_1 = \hat{R}_2$	[tan chord theorem]	✓ S ✓ R
	$\therefore \hat{P}_2 = \hat{A}_2$		
	RP is a tangent to the circle	[converse tan chord theorem] **OR** [∠ between line and chord] **OR** [converse alt seg theorem]	✓ R
			(6)
	OR		
	In ΔMSP and ΔMPA		
	$\hat{M}_2$ is common		✓ S
	AR = AP	[tans from same point]	✓ S / R
	$\hat{R}_1 + \hat{R}_2 = \hat{P}_1 + \hat{P}_2$	[∠s opp equal sides]	✓ S
	$\hat{S}_4 = \hat{R}_1 + \hat{R}_2$	[proved in 10.2]	
	$\therefore \hat{S}_4 = \hat{P}_1 + \hat{P}_2$		✓ S
	$\therefore \hat{P}_2 = \hat{A}_2$	[sum of ∠s in Δ]	✓ S
	RP is a tangent to the circle	[converse tan chord theorem]	✓ R
			(6)
			[15]