5. FUNCTIONS

- ~ Straight Lines
- ~ Parabolas
- ~ Hyperbolas
- ~ Exponential Graphs
- ~ Average Gradient

Trig Graphs:

- ~ Recap
- ~ Amplitude changes
- ~ Period changes
- ~ Horizontal shifts

STRAIGHT LINES

$$y = a x + q$$

a is the gradient

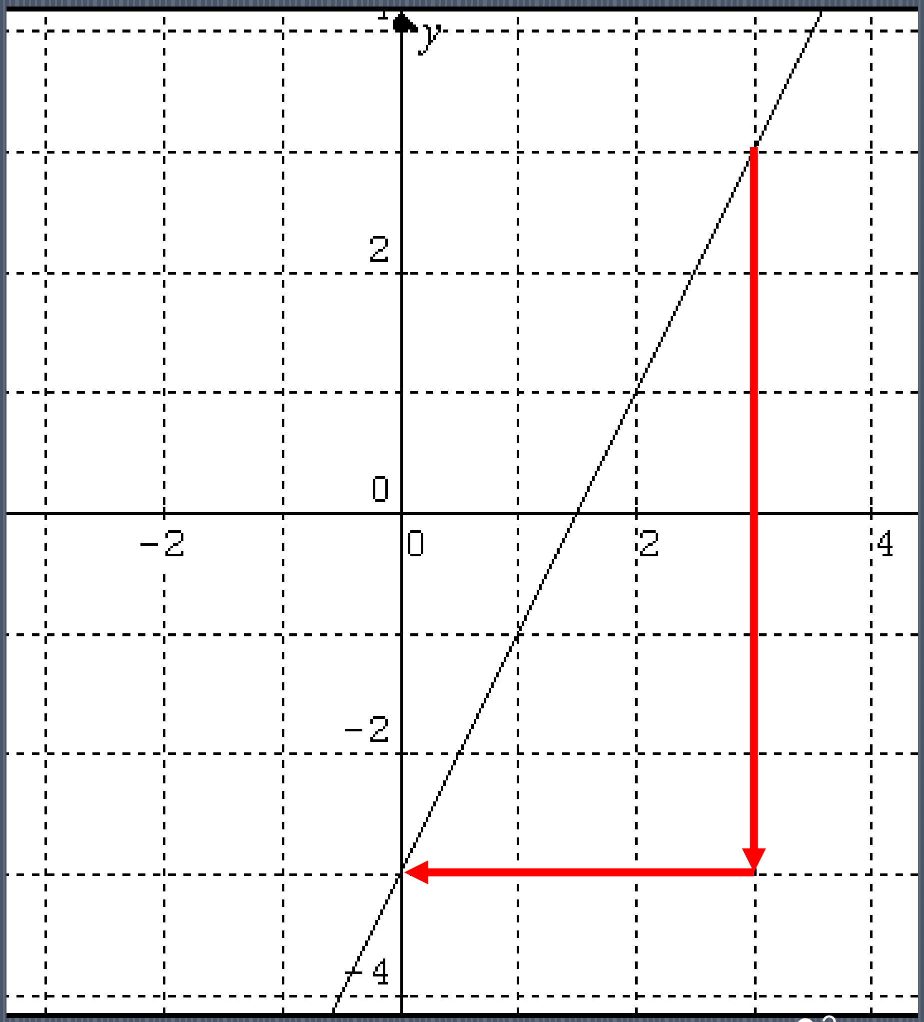
q is the y - intercept

Investigating a and q of Straight Line Graphs

Example

Find the equation of this straight line graph.

- the y intercept is – 3
  so **q** = -3
- the gradient is
  6 down and 3 to the left so **a** = $\frac{-6}{-3}$ = 2
- y = **2**x **– 3**



Finding the Equation of a Straight Line Graph

Practice Finding Equations of Straight Line Graphs

Example

Find the equation of this straight line graph.

$$\mathbf{a} = \frac{y_2 - y_1}{x_2 - x_1}$$

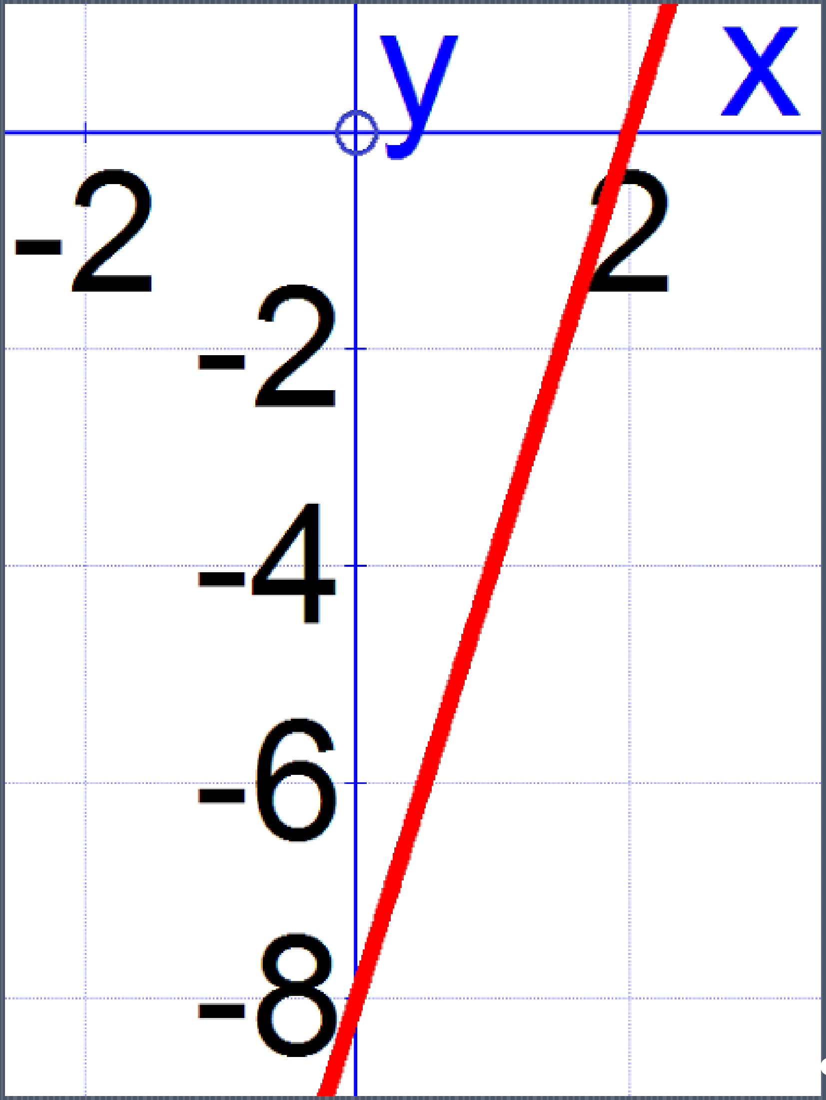
$$= \frac{-8-0}{0-2}$$

$$= \frac{-8}{-2}$$

$$= 4$$

y-intercept = -8

y = 4x – 8



Example

Sketch the graph of y = -3x – 3.

- q = -3 so the y - intercept is – 3
- x –intercept (y=0)

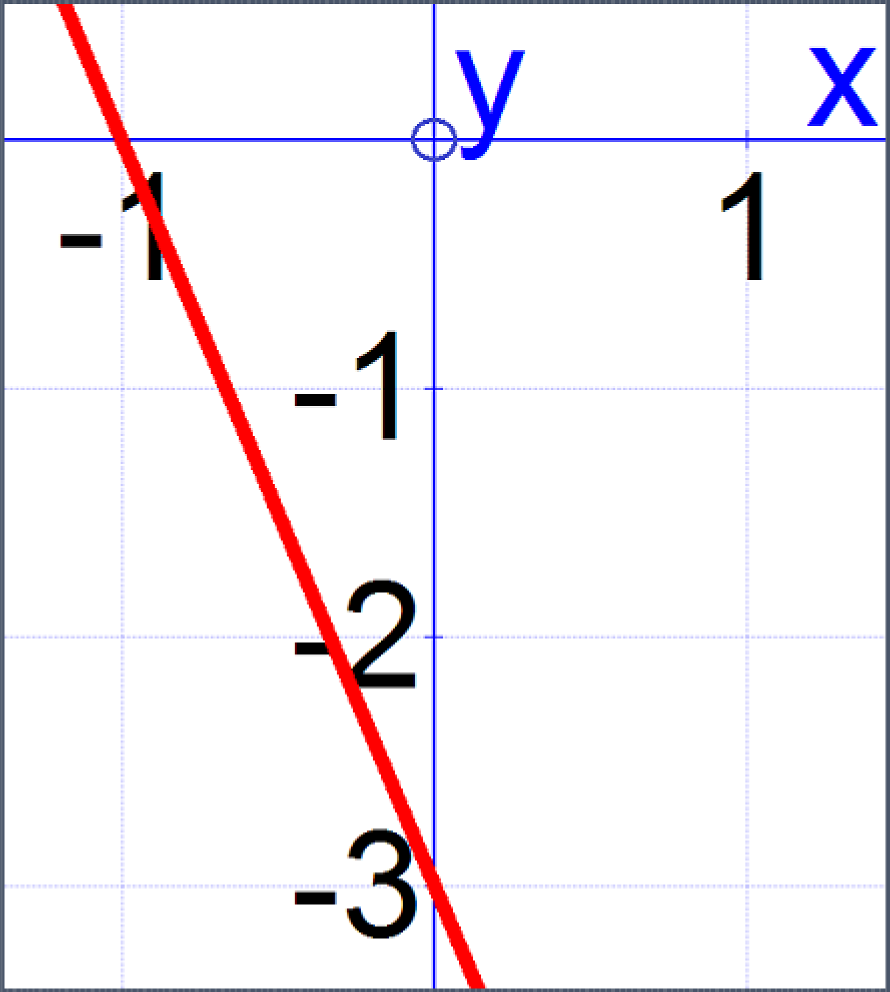
$0 = -3x - 3$

$3x = -3$

$x = -1$

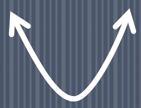
Sketching Straight Line Graphs

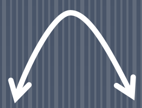
Straight Line Graphs Problems



PARABOLAS

Standard form: $y = ax^2 + bx + c$

a: a > 0: arms go up (smile) 

a < 0: arms go down (frown) 

b: b > 0: graph shifts to the left

b < 0: graph shifts to the right

Investigating the effects of *b* in a Parabola

c: c > 0: positive *y*-intercept

c < 0: negative *y*-intercept

Finding the roots and vertex of a parabola

Effects of a, b & c of Standard form of a Parabola

Example

Sketch the graph of $y = 2x^2 + 5x + 2$

y-intercept: read off std. form

$c = 2$

x-intercepts: $y = 0$

$y = 2x^2 + 5x + 2$

$0 = (2x + 1)(x + 2)$

$x = -\frac{1}{2}$ or $x = -2$

Turning Point: Use formula …

… Sketch: $y = 2x^2 + 5x + 2$ …

$$x = \frac{-b}{2a}$$

(Turning-point formula)

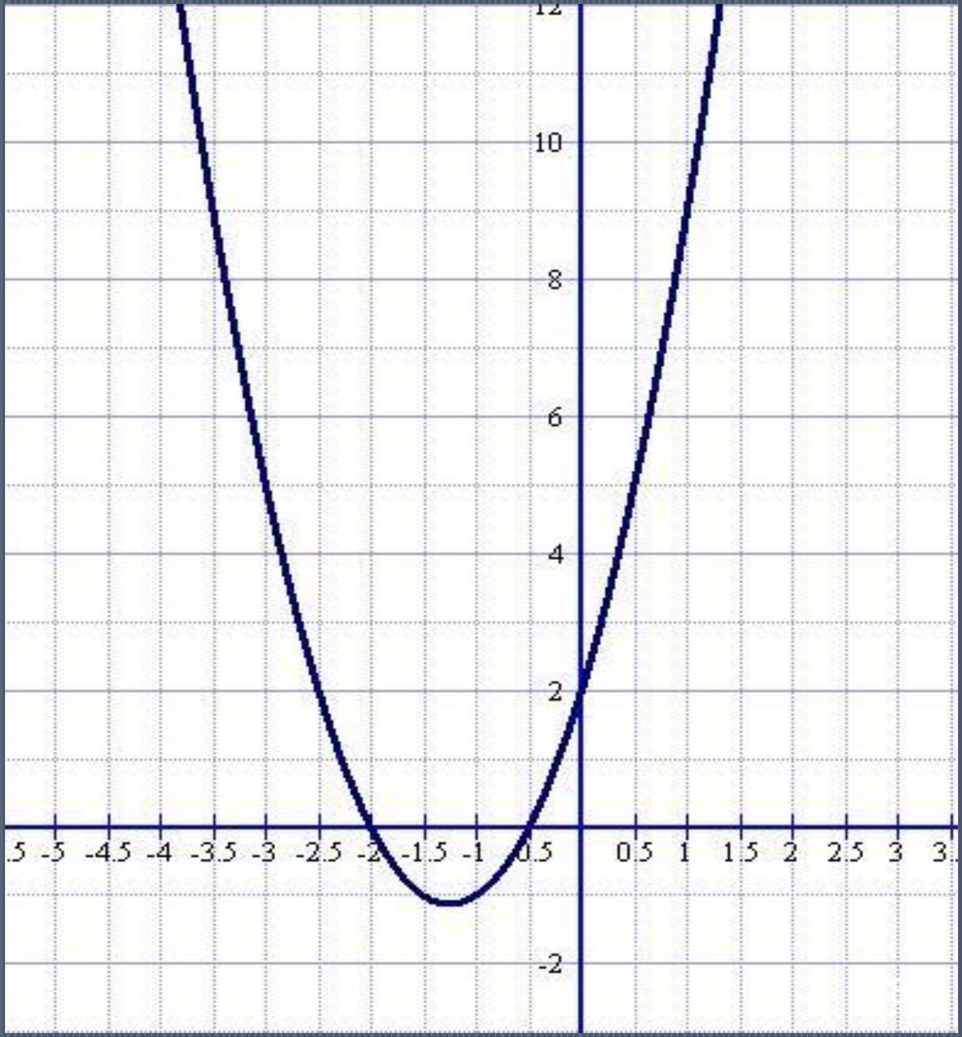
$$x = \frac{-(5)}{2(2)}$$

$$x = \frac{-5}{4}$$

$$y = 2\left(\frac{-5}{4}\right)^2 + 5\left(\frac{-5}{4}\right) + 2$$

$$y = \frac{-9}{8}$$

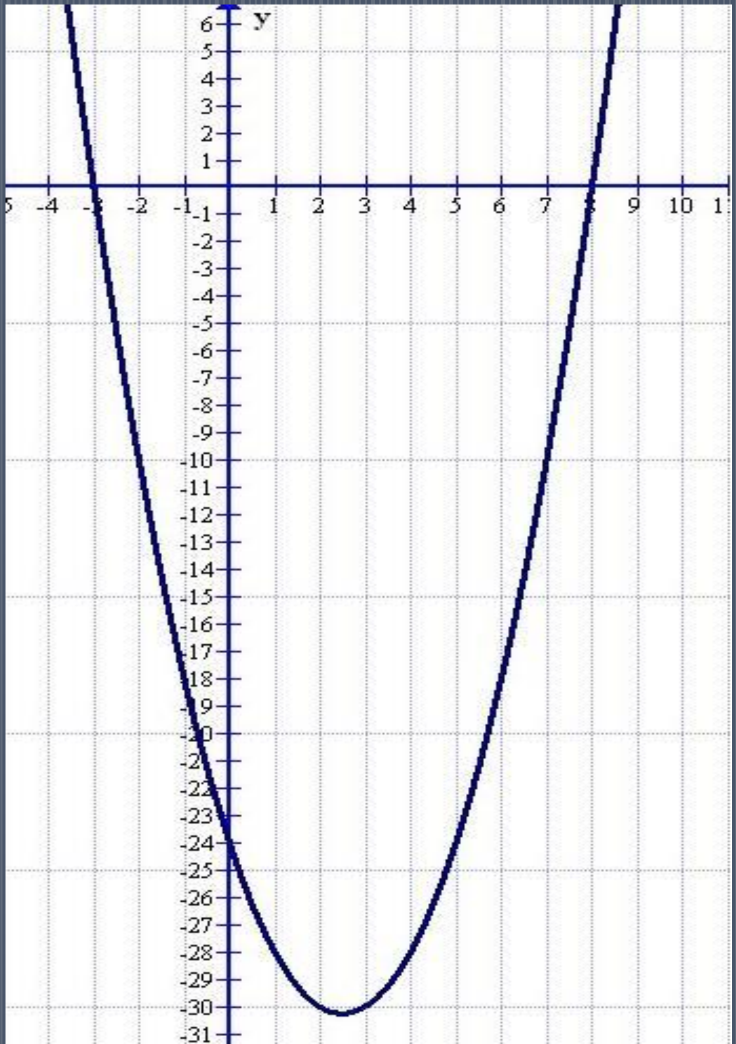
- TP: $\left(\frac{-5}{4};\frac{-9}{8}\right)$



Example

Find the equation of the parabola.

(Given x-intercepts and 1 other point)



$y = a(x - root_1)(x - root_2)$

$y = a(x - (-3))(x - 8)$

$y = a(x + 3)(x - 8)$

Subst. pt: y-int (0;-24)

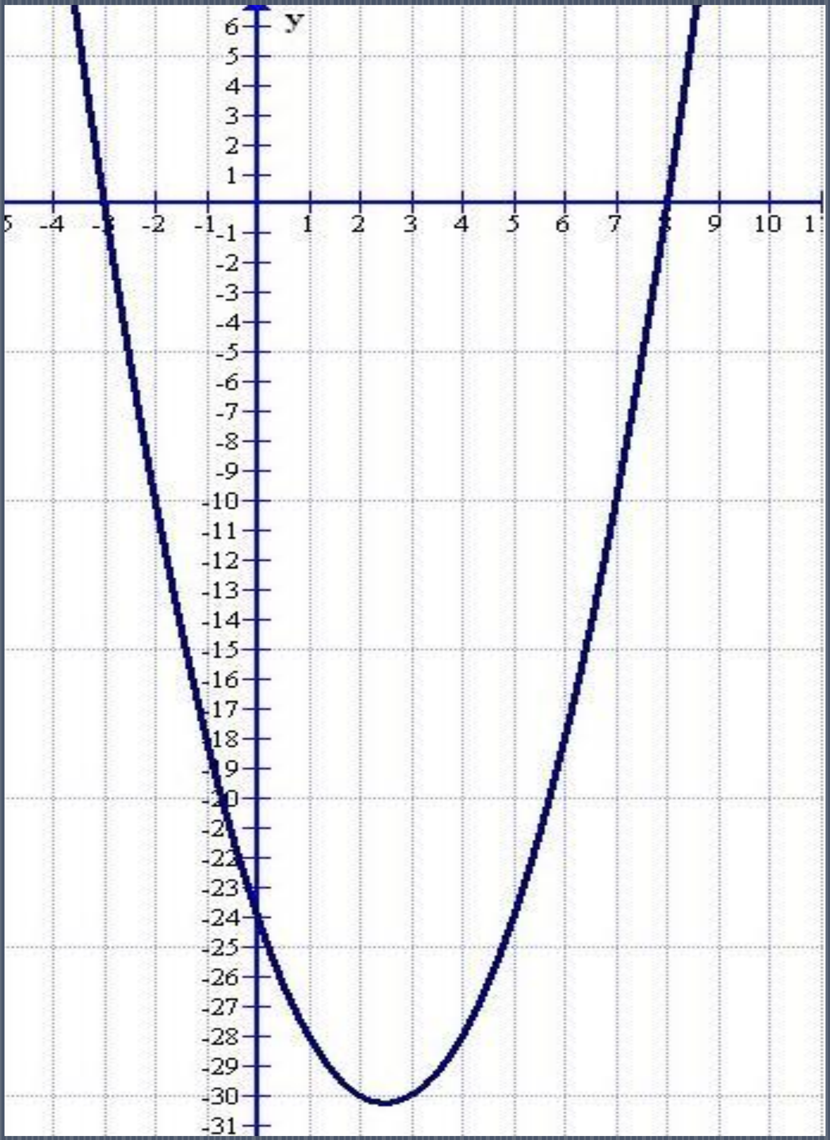
$y = a(x + 3)(x - 8)$

$-24 = a(0 + 3)(0 - 8)$

$-24 = -24a$

$1 = a$

… Find the equation …



Found …

$y = a(x + 3)(x – 8)$

$a = 1$

Equation in Std. Form:

$y = a(x + 3)(x – 8)$

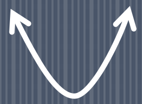
$y = 1(x + 3)(x – 8)$

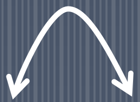
$y = x^2 - 5x - 24$

Finding the Equation of a Parabola

PARABOLAS

Turning - point form: $y = a(x + p)^2 + q$

a: a > 0: arms go up (smile) 

a < 0: arms go down (frown) 

(-p;q) is the co-ordinate of the Turning Point

p: p > 0: graph shifts to the left

p < 0: graph shifts to the right

q: q > 0: graph shifts up

q < 0: graph shifts down

Effects of *p* of the Turning Point form of a Parabola

Example

Sketch the graph of $y = 2(x - 1)^2 - 18$

y-intercept: $x = 0$

$y = 2(0 - 1)^2 - 18$

$y = -8$

TP $(-p; q)$

TP $[-(-1); -18]$

TP $(1; -18)$

x-intercepts: $y = 0$

$y = 2(x - 1)^2 - 18$

$0 = 2(x^2 - 2x + 1) - 18$

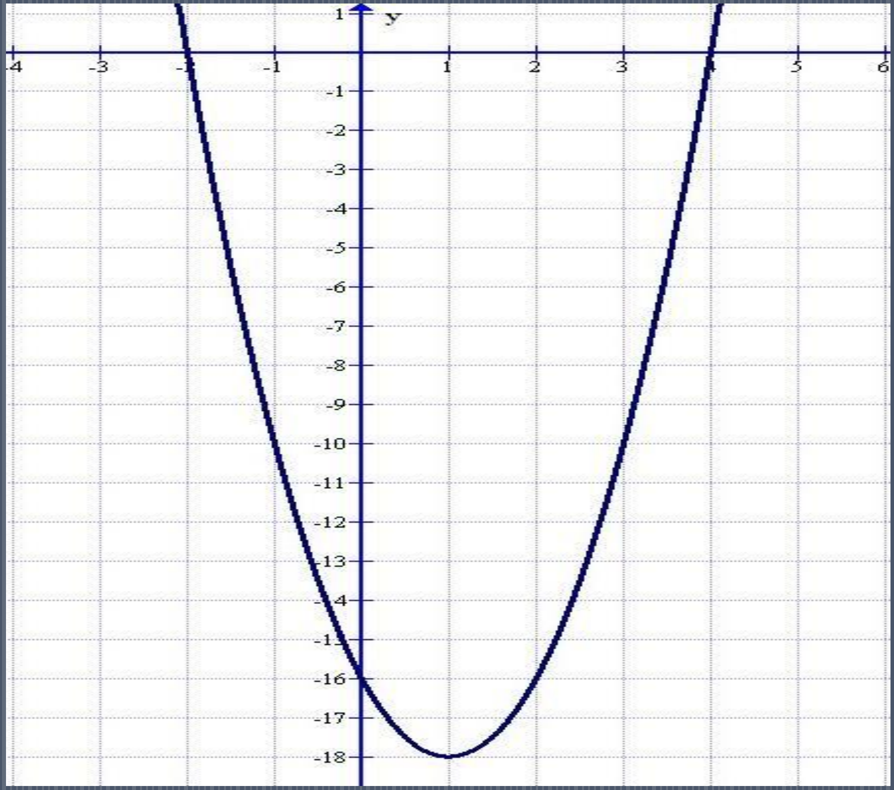
$0 = 2x^2 - 4x + 2 - 18$

$0 = 2x^2 - 4x - 16$

$0 = x^2 - 2x - 8$

$0 = (x - 4)(x + 2)$

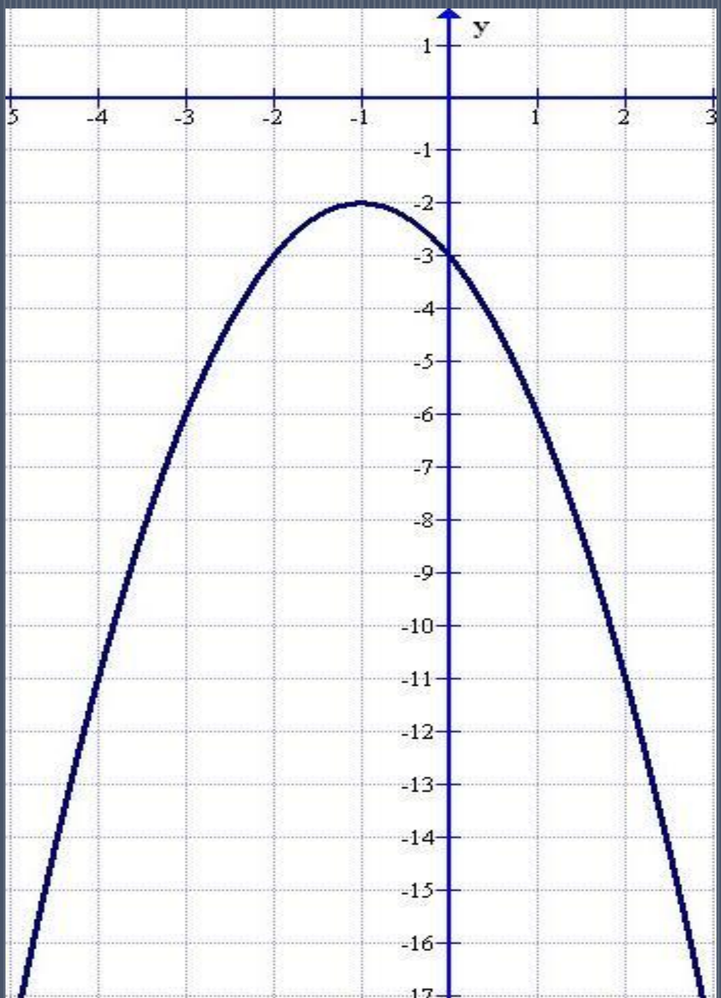
$x = 4$ or $x = -2$



Example

Find the equation of the parabola.

(Given the turning-point and 1 other point)



TP (-p;q) $\therefore$ TP [-(1);-2]

$$y = a(x + p)^2 + q$$

$$y = a(x + 1)^2 - 2$$

Subst. pt: y-int (0;-3)

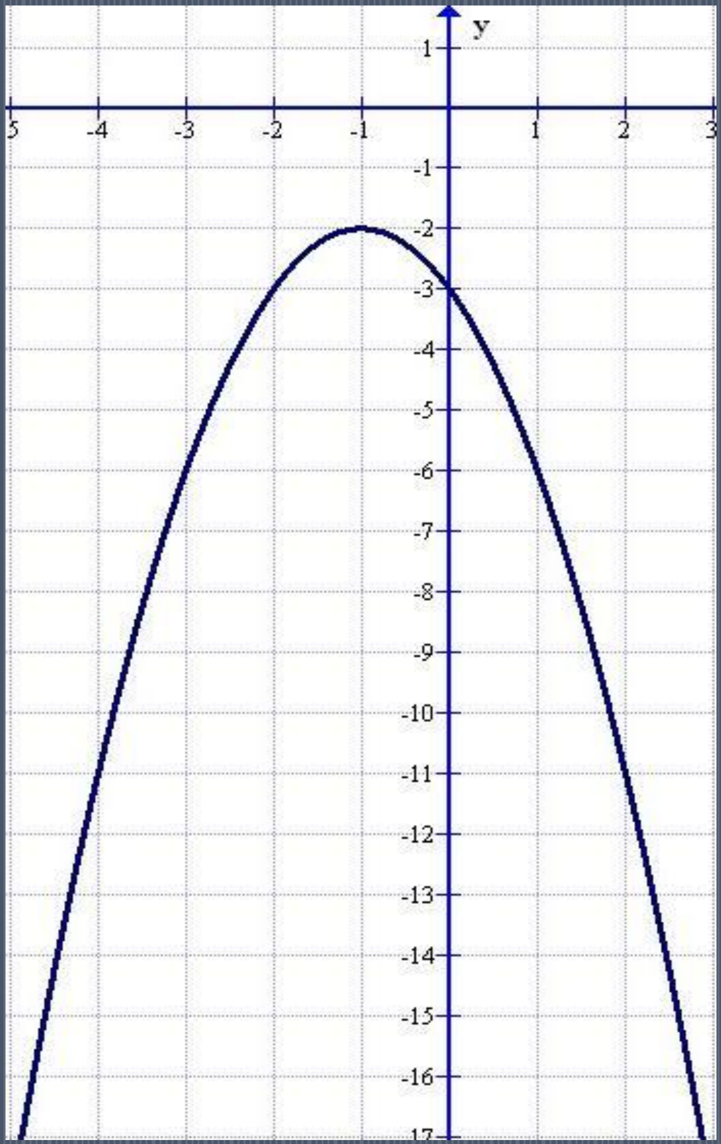
$$y = a(x + 1)^2 - 2$$

$$-3 = a(0 + 1)^2 - 2$$

$$-3 = a - 2$$

$$-1 = a$$

… Find the equation …



Found …

$y = a(x + 1)^2 - 2$

$a = -1$

Equation in Std. form:

$y = a(x + 1)^2 - 2$

$y = -1(x + 1)^2 - 2$

$y = -x^2 - 2x - 3$

[Finding the Turning Point Formula of a Parabola]

REFLECTING PARABOLAS

Example: $y = x^2 + 8x - 2$

*** Reflect in the x-axis: every y swops signs**

$-y = x^2 + 8x - 2$

$y = -x^2 - 8x + 2$

*** Reflect in the y-axis: every x swops signs**

$y = (-x)^2 + 8(-x) - 2$

$y = x^2 - 8x - 2$

*** Reflect in the line y = x: swop x and y**

$x = y^2 + 8y - 2$

Reflecting Parabolas

Reflecting Lines

Parabolas in the Real World

[Parabolic Mirrors]

[Graphical Representation of a Projectile]

Revision: Graphs

[Match the Equation to the Parabola]

[Parabola Roots Problems]

[Higher-Order Functions Problems (start 1:35 - end 5:45)]

HYPERBOLAS

RECAP! $y = \frac{a}{x} + q$

Sketch the following graphs and write down the equation of the asymptotes:

1) $y = \frac{2}{x} - 1$

2) $y = \frac{-2}{x} + 4$

What are the equations of the asymptotes for …

1) Vertical asymptote: x = 0
   Horizontal asymptote: y = -1
2) Vertical asymptote: x = 0
   Horizontal asymptote: y = 4

Standard form of a Hyperbola:

$$y = \frac{a}{x+p} + q$$

- **a** determines the quadrants

  $a > 0$ => Q 1 & 3

  $a < 0$ => Q 2 & 4

- **q** determines the horizontal asymptote

  i.e. vertical translation OR up/down shifts

  $q > 0$ => graph shifted up

  $q < 0$ => graph shifted down

Standard form of a Hyperbola:

$$y = \frac{a}{x+p} + q$$

- **p** determines the vertical asymptote

  i.e. horizontal translation OR left/right shifts

  $p > 0$ => graph shifted left

  $p < 0$ => graph shifted right

Example:

Sketch the graph of: $y = \frac{3}{x-3} + 4$

- a (quadrants):

  Q 1 & 3

- q (horizontal asymptote):

  y = 4

- p (vertical asymptote):

  p<0 so graph shifted to the right

  $x = 3$

- Now, what about the x- and y-intercepts?

- y – intercept (x=0):

$$y = \frac{3}{0-3} + 4$$

$$= \frac{3}{-3} + 4$$

$$= 3$$

- x – intercept (y=0):

$$0 = \frac{3}{x-3} + 4$$

$$-4 = \frac{3}{x-3}$$

$$-4(x-3) = 3$$

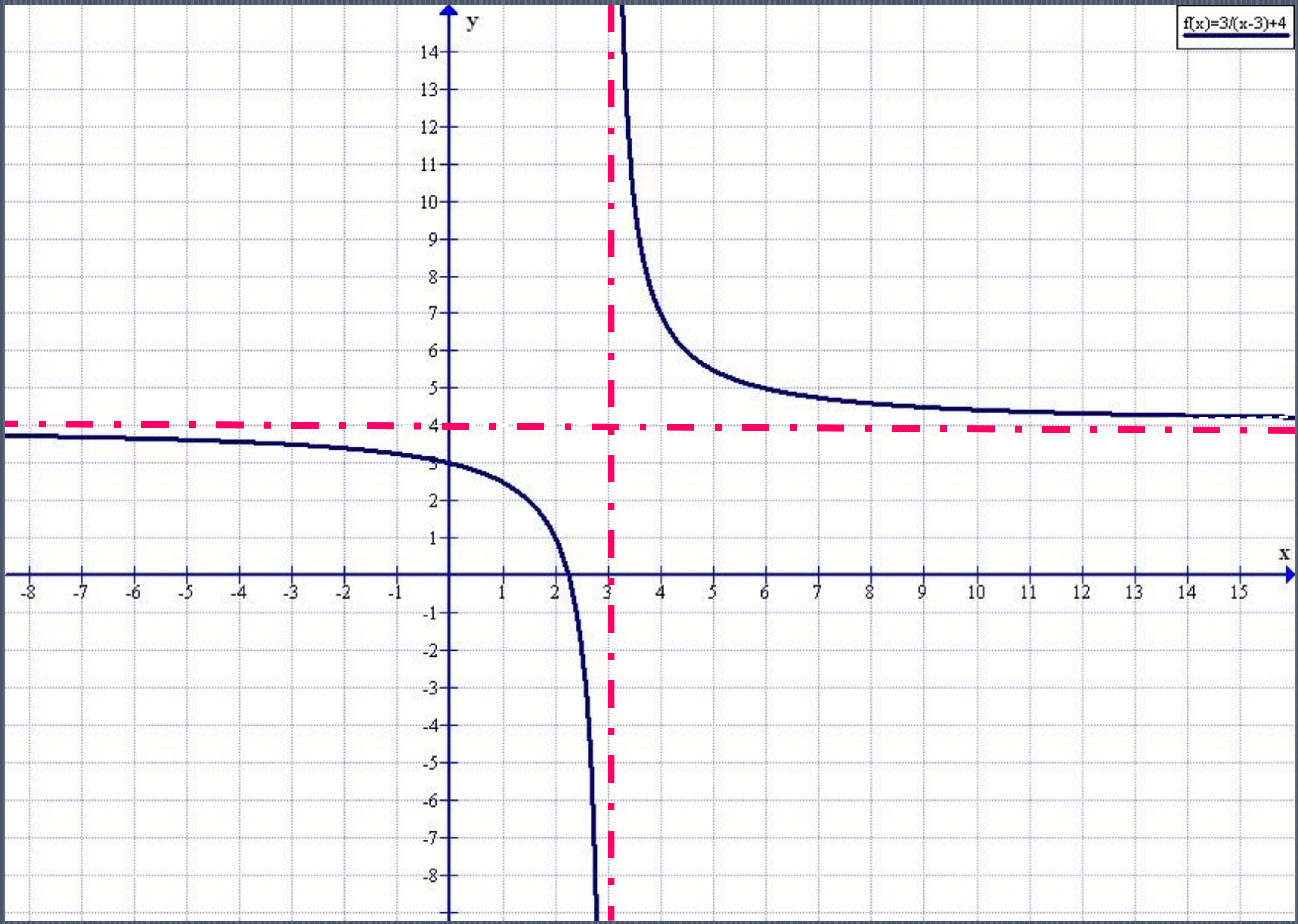
$$-4x + 12 = 3$$

$$-4x = -9$$

$$x = 2{,}25$$

$$y = \frac{3}{x-3} + 4$$

Sketching Hyperbolas



Example:

Sketch the graph of: $y = \frac{-2}{x+3} - 4$

- a (quadrants):

  Q 2 & 4

- q (horizontal asymptote):

  y = -4

- p (vertical asymptote):

  p>0 so graph shifted to the left

  x = -3

- Now, what about the x- and y-intercepts?

- y – intercept (x=0):

$$y = \frac{-2}{0+3} - 4$$

$$= \frac{-2}{3} - 4$$

$$= 4{,}67$$

- x – intercept (y=0):

$$0 = \frac{-2}{x+3} - 4$$

$$4 = \frac{-2}{x+3}$$

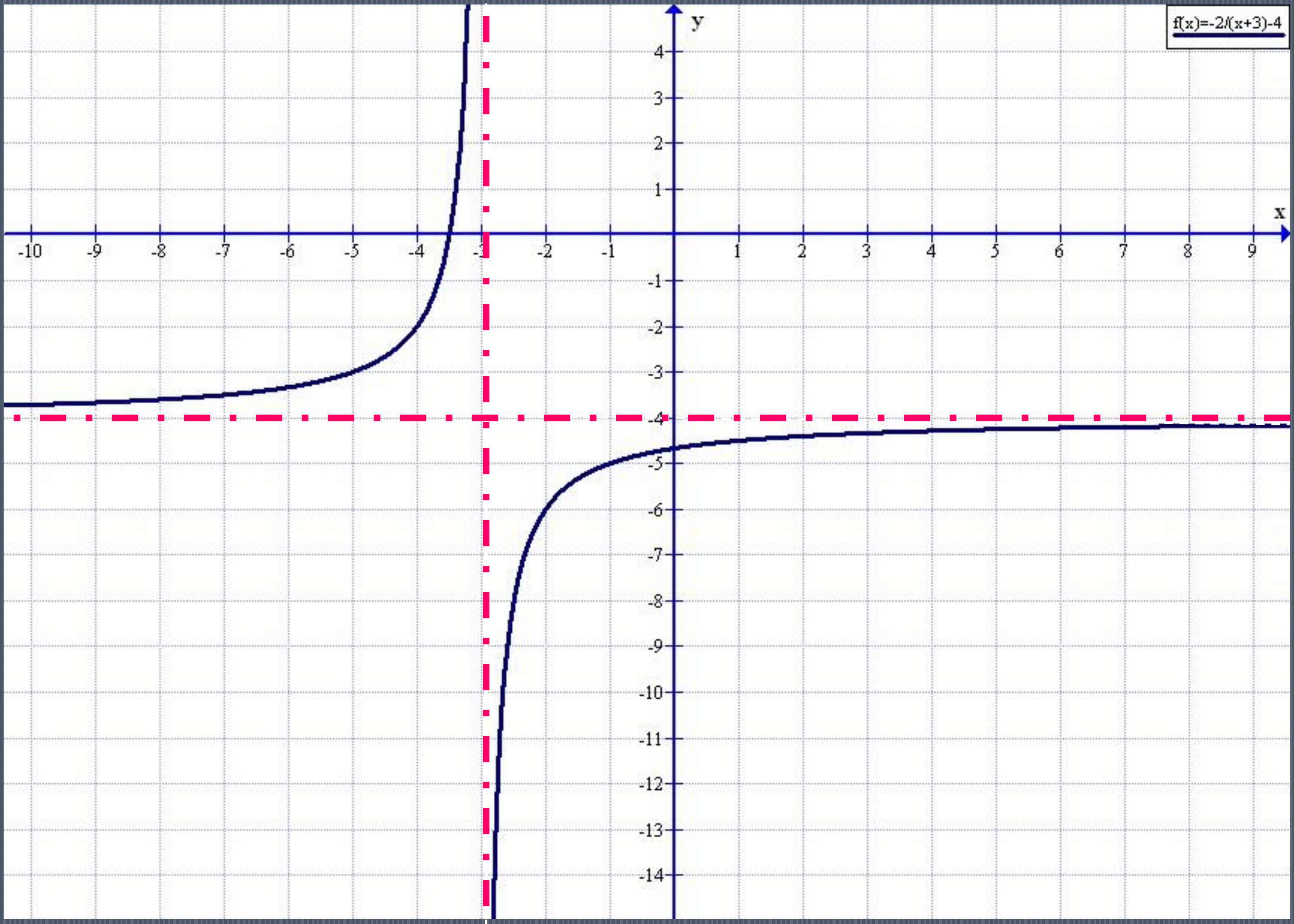
$$4(x+3) = -2$$

$$4x + 12 = -2$$

$$4x = -14$$

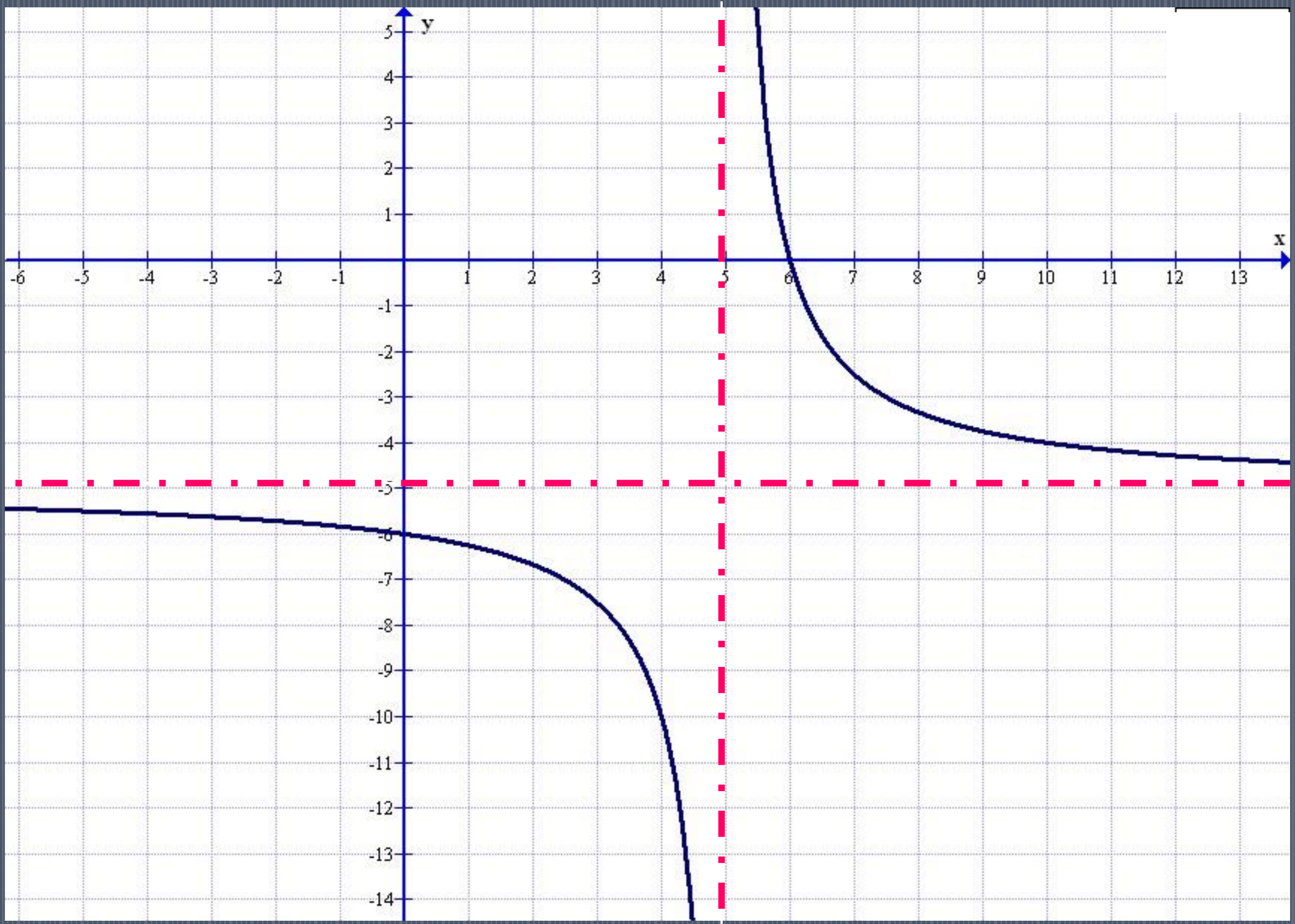
$$x = -3{,}5$$

$$y = \frac{-2}{x+3} - 4$$



Example:

Find the equation of the following graph:



$$\boldsymbol{y = \frac{a}{x+p} + q}$$

- Substitute in the asymptotes ...

$$y = \frac{a}{x-5} - 5$$

- Substitute a point that lies on the graph ...

Subst: (6;0)

$$0 = \frac{a}{6-5} - 5$$

$$5 = \frac{a}{1}$$

$$5 = a$$

$$y = \frac{5}{x-5} - 5$$

The Hyperbola

EXPONENTIAL GRAPHS

RECAP! $y = a.b^x + q$

Sketch the following graphs and write down the equation of the asymptote:

1) $y = 5^x$ 2) $y = 5^{-x} + 2$ 3) $y = \frac{1}{2}^x - 2$

What is the equation of the asymptote for …

1) $y = 0$ (i.e. x-axis)
2) $y = 2$
3) $y = -2$

Cell Division

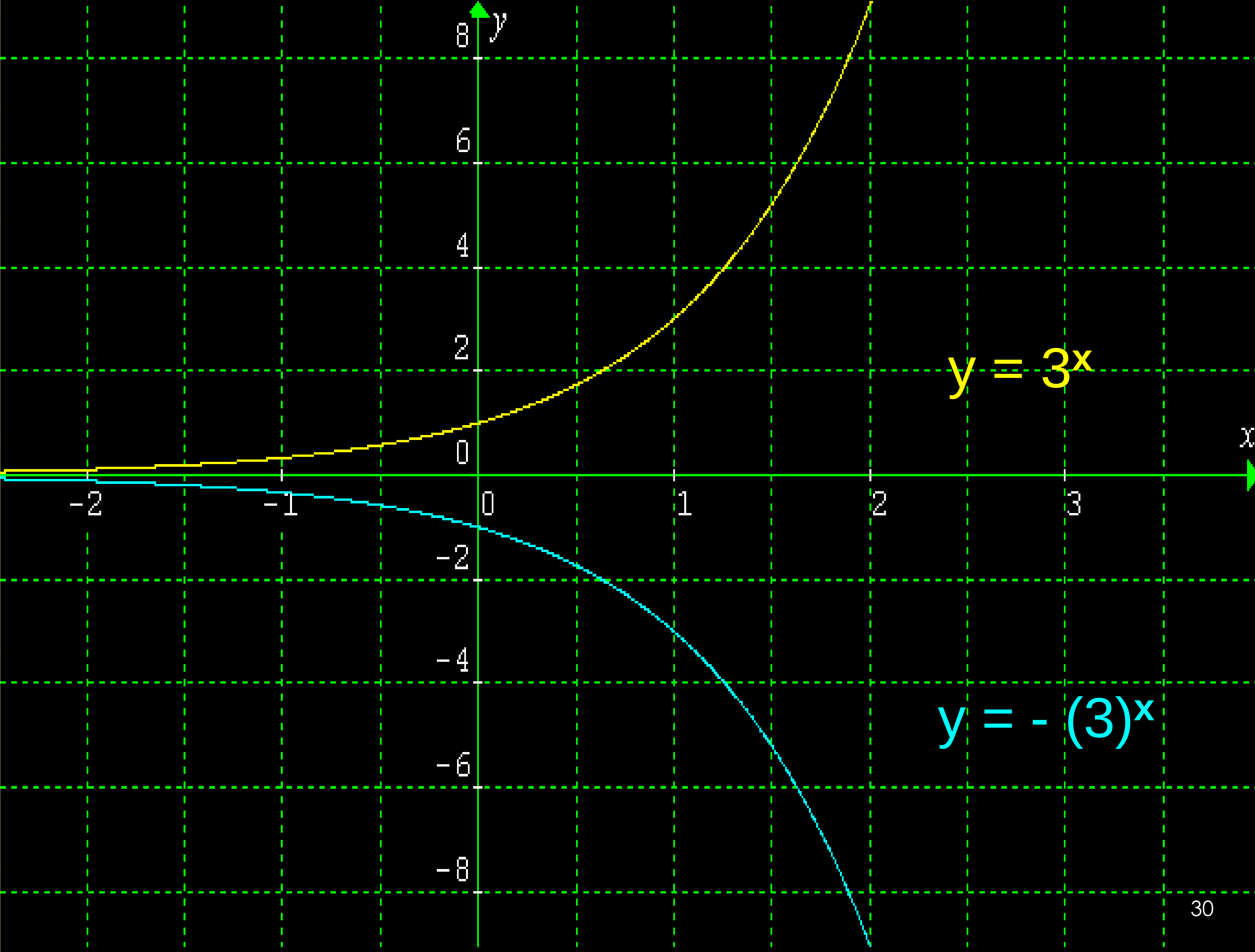
Example:

Sketch the graph of: $y = -(3)^x$

How will it differ from $y = 3^x$?

What are the intercepts?

What is the equation of the asymptote ?


y
8
6
4
2
0
x
-2
-1
0
1
2
3
-2
-4
-6
-8
y = 3^x
y = - (3)^x

Standard form of an Exponential:

$$y = a.bx^{+p} + q$$

- **q** is the horizontal asymptote

  i.e. represents a vertical shift (up/down shift)

  $q > 0$ => graph shifted up

  $q < 0$ => graph shifted down

- **p** represents a horizontal shift (left/right shift)

  $p > 0$ => graph shifted left

  $p < 0$ => graph shifted right

Standard form of an Exponential:

$$y = a.b^{x+p} + q$$

- **b** determines the shape of the graph

  b > 0 => increasing function

  0 < b < 0 (a fraction) => decreasing function

- **a** determines where the graph lies

  a > 0 => graph lies above the x-axis

  a < 0 => graph lies below the x-axis

Example:

Sketch the graph of: $y = 5.5^{x} + 1$

Asymptote:

$y = 1$

(no x-intercept)

y-intercept:

$y = 5.5^{0} + 1$

$= 5.1 + 1$

$= 6$

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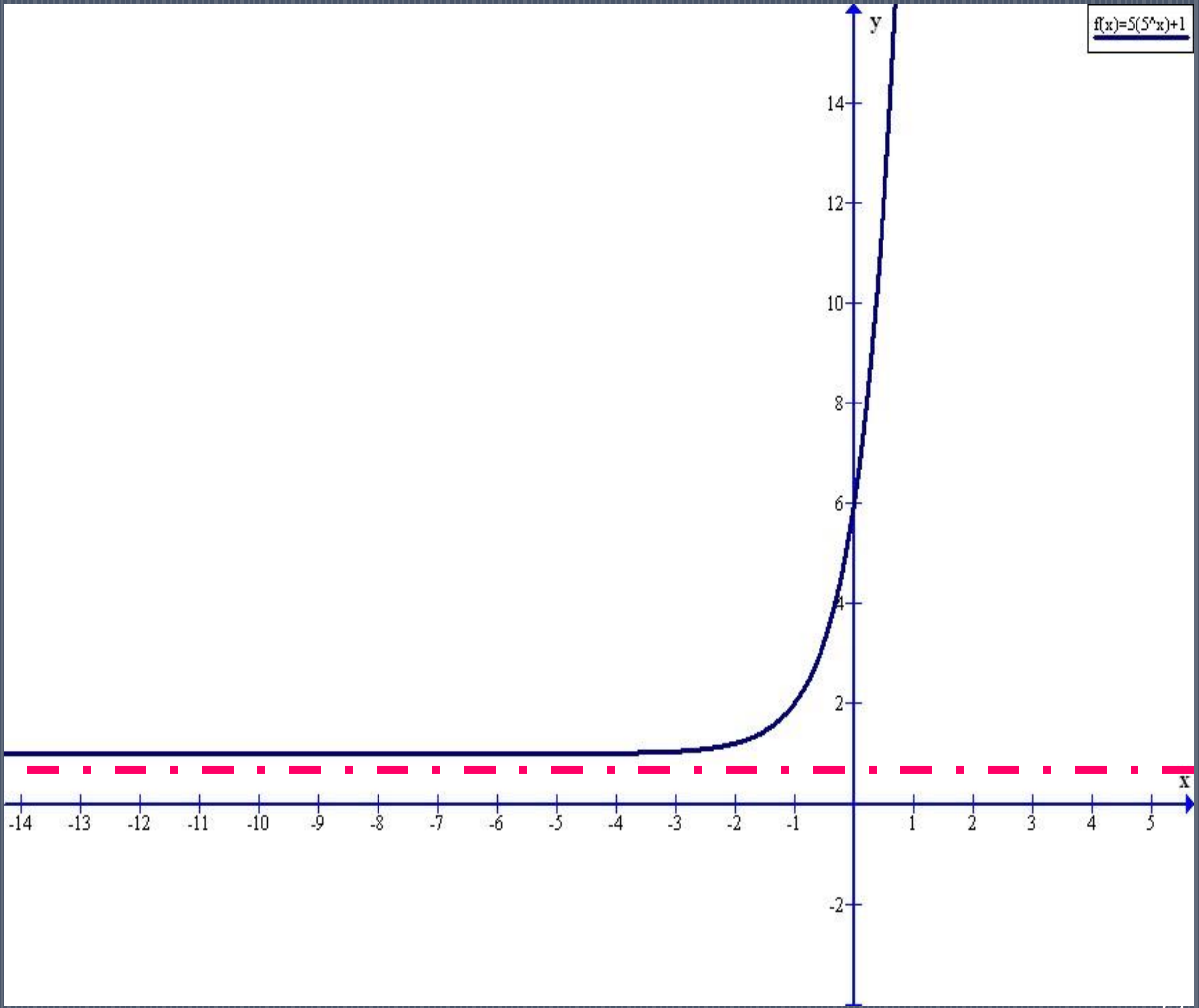

Example:

Sketch the graph of: $y = -3.3^{-x} - 3$

Asymptote:

$y = -3$

(no x-intercept)

y-intercept:

$y = -3.3^{-0} – 3$

$= -3.1 -3$

$= 6$

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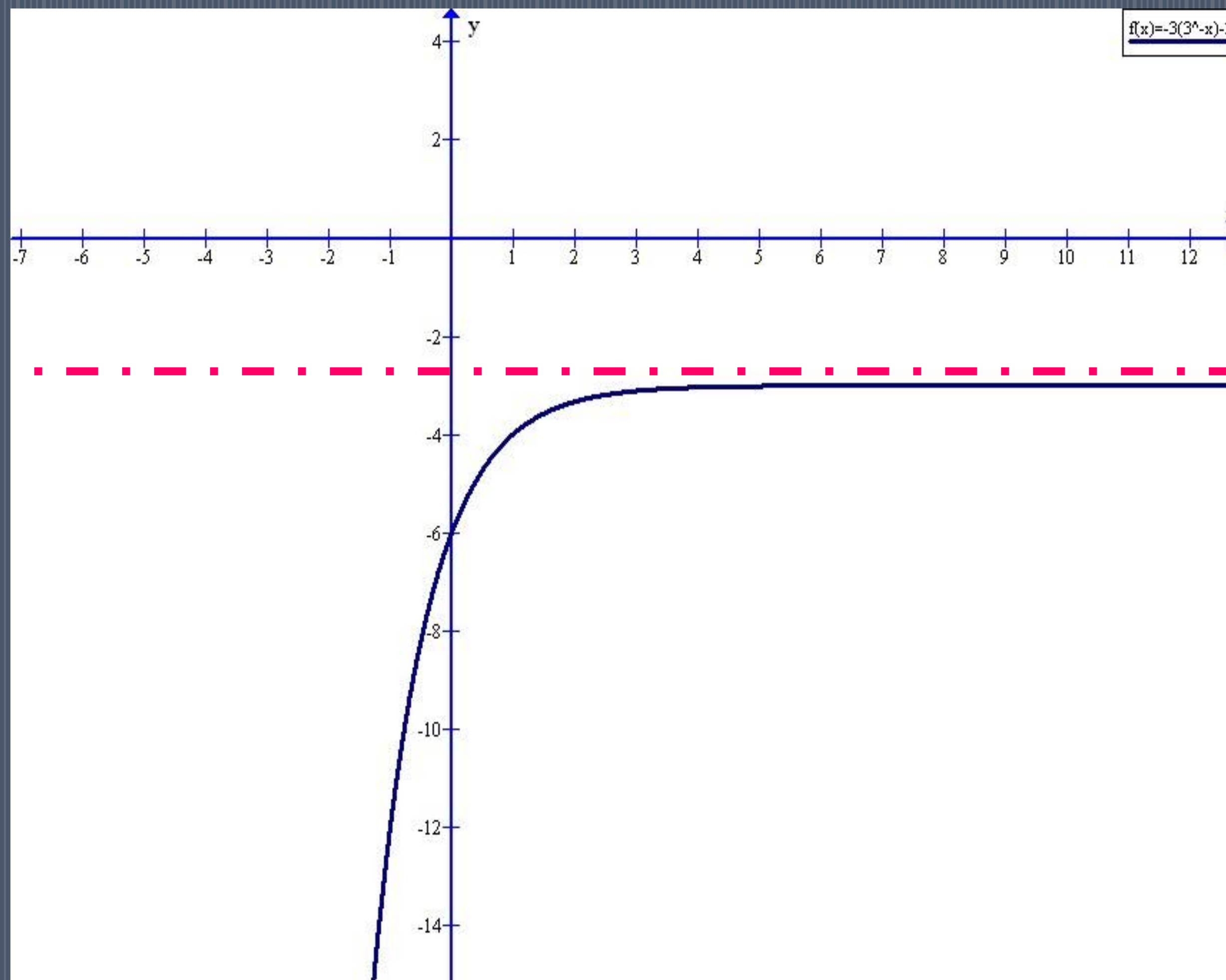


Example:

Sketch the graph of: $y = 4.2^{x+1} - 2$

Asymptote:

$y = -2$

(no y-intercept)

x-intercept:

$0 = 4.2^{x+1} - 2$

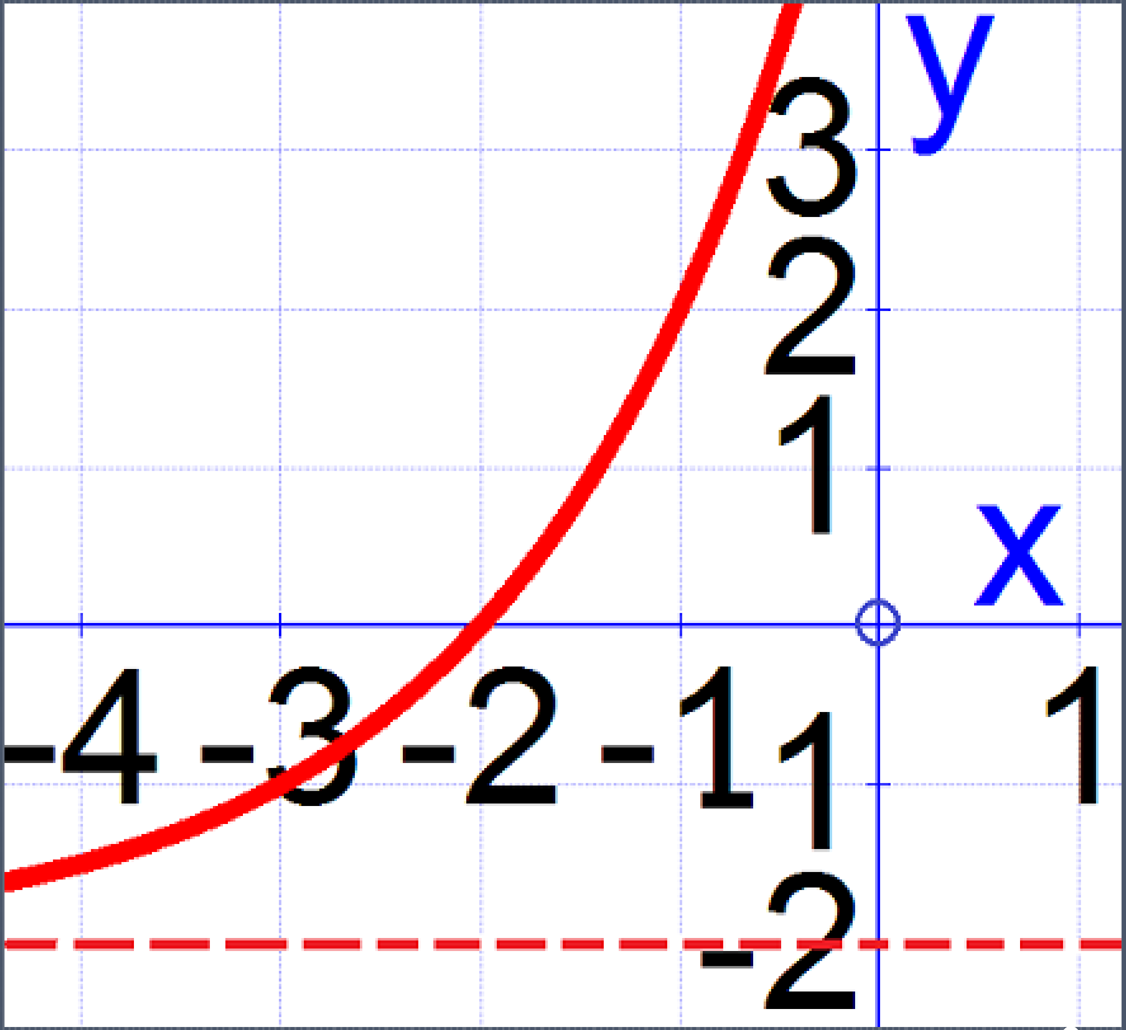
$2 = 4.2^{x+1}$

$\frac{1}{2} = 2^{x+1}$

$2^{-1} = 2^{x+1}$

$-1 = x + 1$

$x = -2$



Example:

Find the equation of the graph if $y = a.b^x + q$:

Subst. in asymptote:

$y = a.b^x + 1$

Subst. y-intercept:

$3 = a.b^0 + 1$

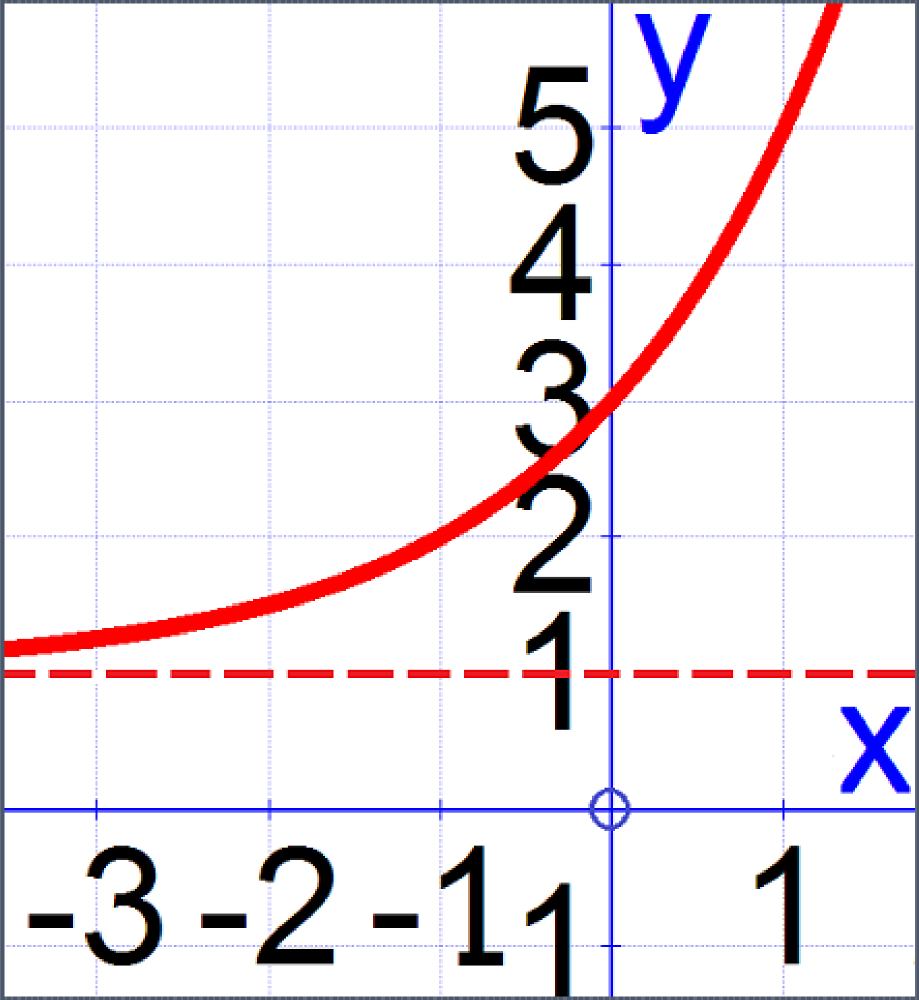
$2 = a$

Subst. in pt (1;5):

$5 = 2.b^1 + 1$

$4 = 2b$

$b = 2$ **$y = 2.2^x + 1$**

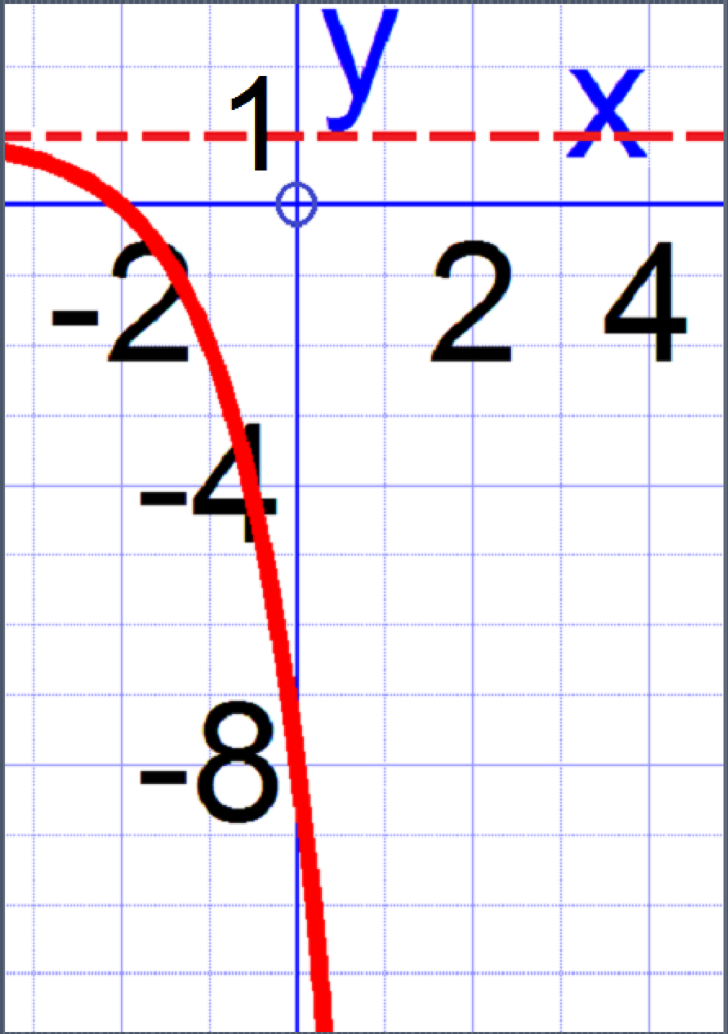


Example:

Summary of Exponential Transformations

Determine the values of a and q, given

$y = a.3^{x+1} + q$:



Subst. in asymptote:

$y = a.3^{x+1} + 1$

$q = 1$

Subst. x-intercept:

$0 = a.3^{-2+1} + 1$

$-1 = a.\ 3^{-1}$

$a = -3$

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Finding the Equations of Exponential Graphs

Exponential Function in Life

- Exponential growth of a bacterial culture

Exponential Growth

- Exponential decay of radioactive material

Exponential Decay