**NEWTON’S SECOND LAW OF MOTION**

**1. A SINGLE OBJECT MOVING ON A HORIZONTAL PLANE WITHOUT FRICTION.**

A 15 kg cement block is pulled across a smooth surface with a force of 100 N, which forms an angle of 14° with the horizontal. Calculate the magnitude of the normal force and the acceleration of the cement block. The effects of friction may be ignored.

**SOLUTION**

**Step 1: Draw a free body diagram**

N F

N

**OR** FV F

H

w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 In this case, the applied force acts at an angle and therefore the normal force is not just equal to the weight of the object. The magnitude of the vertical component of the applied force

together with the magnitude of the normal force equals the magnitude of the weight i.e.

*w = N + Fsin14°.*

 The forces in the vertical plane i.e. the normal force, weight and the vertical component of the applied force do not affect the horizontal motion. Only one force i.e. the horizontal component

of the applied force, influences horizontal motion.

**Step 3: Solve**

Normal force; upwards positive:

w + N + Fv = 0

mg + N + F sin14° = 0

-(15)(9,8) + N + 100sin14° = 0 (Use chosen sign convention when substituting.)

 N = 122,81 N Acceleration:

To the right as positive:

Fnet = ma

Fcos14°= ma

(100)cos14° = 15a

a = 6,47 m∙s-2

a = 6,47 m∙s-2 to the right

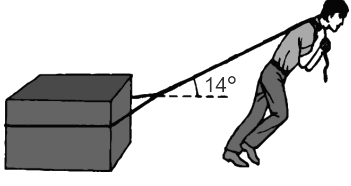
**Step 4: Evaluate/interpret the answer**

The answer is positive – it shows that the acceleration is towards the right.

The normal force is smaller than the weight due to the vertical component of the applied force.

**2. A SINGLE OBJECT MOVING ON A HORIZONTAL PLANE WITH FRICTION.**

A 15 kg cement block is pulled across the floor with a force of



100 N, which forms an angle of 14° with the horizontal. The kinetic friction coefficient between the block and the floor is 0,4.

Calculate the acceleration of the cement block.

**SOLUTION**

**Step 1: Draw a free body diagram**

N F

f OR

N

FV

f FH

w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. *Fnet = ma.*

 The forces in the vertical plane i.e. the normal force, weight and the vertical component of the applied force do not affect the horizontal motion. Only two forces i.e. friction and the

horizontal component of the applied force, influence horizontal motion.

 In this case, the applied force acts at an angle and therefore the normal force is not just equal to the weight. The magnitude of the vertical component of the applied force together with the

magnitude of the normal force equals the magnitude of the weight i.e. *w = N + Fsin14°.*

Although the normal force is not asked in this question, it is needed to calculate the frictional force.

**Step 3: Solve**

To the right as positive: Fnet = ma

FH + f = ma (The net force is the vector sum of all the forces acting on the block.)

Fcos14° + kN = ma

Fcos14° - (0,4)(mg – Fsin14°) = ma (Apply chosen sign convention when substituting.)

100cos14° - (0,4)[(15)(9,8) - 100sin14°] = 15a

a = 3,19 m∙s-2

a = 3,19 m∙s-2 to the right

**Step 4: Evaluate/interpret the answer**

The answer is positive – it shows that the acceleration is towards the right. The acceleration is smaller than in problem1 due to the presence of a frictional force.

**3. A SINGLE OBJECT MOVING ON AN INCLINED PLANE WITHOUT FRICTION.**

An inclined surface is at an angle of 35° to the horizontal. Due to an applied force, F, parallel to the surface, the object of mass 12 kg accelerates at 1,5 m∙s-2. Ignoring all frictional forces, calculate the magnitude and direction of F if the:

3.1 Acceleration is upwards, along the surface

3.2 Acceleration is downward, along the surface

**SOLUTION**

**Step 1: Draw a free body diagram**

12 kg

35°

3.1 To accelerate upwards, the applied force should act upwards along the inclined plane.

F

N F N

OR

w w 

w//

3.2 As above - to have an acceleration smaller than gsin35°, the applied force should act upwards along the inclined plane.

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. *Fnet = ma.*

 Only the forces or component of forces parallel to the incline will influence motion along the incline i.e. F and mgsin35°.

 In Q3.1, the direction of the acceleration is opposite to that of the component of weight down the incline. In Q3.2 the direction of the acceleration is the same as that of the component of

weight down the incline.

**Step 3: Solve**

Q3.1:

Upwards along the incline as positive:

Fnet = ma

F + w// = ma (The net force is the vector sum of all the forces acting on the object.)

F + mgsin35° = ma

F - (12)(9,8)sin35° = (12)(1,5) (Apply chosen sign convention when substituting.)

 F = 85,45 N

 F = 85,45 N upwards along the incline

Q3.2:

Upwards along the incline as positive:

Fnet = ma

F + w = ma

F + mgsin35° = ma

F - (12)(9,8)sin35° = (12)(-1,5) (Apply chosen sign convention when substituting.)

 F = 49,45 N

 F = 49,45 N upwards along the incline

**Step 4: Evaluate/interpret the answer**

Both answers are positive as expected – it shows that the force in both cases acts upwards parallel to the inclined plane.

**4. A SINGLE OBJECT MOVING ON AN INCLINED PLANE WITH FRICTION.**

Richard pulls a crate of mass 20 kg with the help of a rope up along an inclined plane as shown. The tension in the rope is 147 N and the coefficient of kinetic friction between the crate and the inclined plane is 0,1 while it moves up the inclined plane. Calculate the acceleration of the block.

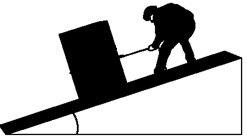
**SOLUTION**

**Step 1: Draw a free body diagram**

N

30°

147 N



F N F

**OR**

f f

w w// w 

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 The normal force is needed to calculate the frictional force. The magnitude of the normal

force is equal to the magnitude of the component of weight perpendicular to the inclined plane i.e. mgcos30°.

 Three forces will influence the motion on the inclined plane i.e. f, F and w//.

**Step 3: Solve**

Upward along the incline as positive: Fnet = ma

F + f + w// = ma (The net force is the vector sum of all the forces acting on the block.) F + kN + w// = ma

F + k w  + w// = ma

F + k mgcos30° + mgsin30° = ma

147 - (0,1)(20)(9,8)cos30° – (20)(9,8)sin30° = 20a

a = 1,60 m∙s-2

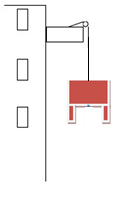
a = 1,60 m∙s-2 upwards along the incline

**Step 4: Evaluate the answer**

As expected the answer is positive i.e. the direction of motion upwards along the incline.

**5. A SINGLE OBJECT MOVING IN THE VERTICAL PLANE.**

A moving company needs to lift a 320 kg piano to the top floor of an apartment building. They set up a rope and pulley system on the balcony of the upper story apartment, and pull



the piano up. If the piano initially has an acceleration of 0,45 m.s-2, what is the

tension in the rope during that period of time?

**SOLUTION**

**Step 1: Draw a free body diagram** T

w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 No normal force is included in the free body diagram. The normal force is a force due to the contact between two surfaces; hence a normal force is a contact force. The piano is not

resting on any surface – therefore there is no normal force.

**Step 3: Solve** Upward as positive: Fnet = ma

T + mg = ma (The net force is the vector sum of all the forces acting on the piano.)

T – (320)(9,8) = (320)(0,45) (Apply chosen sign convention when substituting.) T – 3 100 = 140

T = 3 280 N (positive sign)

T = 3 280 N upwards

**Step 4: Evaluate the answer**

In this problem, the rope exerts an upwards force on the piano. The force by the rope needs to be strong enough to both support the piano against the force of gravity (3 136 N) and to give it an upwards acceleration. So tension needs to be greater than 3 136 N.

**6. TWO-BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, BOTH ON A FLAT HORIZONTAL PLANE WITHOUT FRICTION.**

Three blocks of masses 1 kg, 2 kg and 3 kg moves on a frictionless horizontal surface under the

influence of a force of 42 N as shown.

Calculate the:

1 kg

2 kg

3 kg

42 N

6.1 Acceleration of the system

6.2 Tension in rope joining the 1 kg and the 3 kg blocks

6.3 Force exerted by the 1 kg block on the 2 kg block

**SOLUTION**

**Step 1: Draw a free body diagram** for each block.

1 kg: N

F21 T

2 kg:

N

F12

3 kg: N T

42 N

w w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 To calculate the acceleration, the whole system can be considered as a unit**. (Previously it was not accepted, but there is however, no problem to consider the system as a unit**

**when calculating the acceleration as the magnitude (and even the direction) of the**

**acceleration is the same for both objects. However, when calculating the tension in the rope it is needed to consider the free body diagram of each object separately.)**

 In the absence of a frictional force, the applied force is the net force acting on the system.

 To find the tension in the rope and the force exerted by the 1 kg block on the 2 kg block, each block should be isolated and Newton’s second law should be applied to each block

separately.

**Step 3: Solve**

To the right as positive:

6.1 Fnet = ma

42 = (2 + 1 + 3)a a = 7 m.s-2

a = 7 m∙s-2 to the right

6.2 Consider the free body diagram of the 3 kg block; to the right as positive:

Fnet = ma

T + F = ma

T + 42 = (3)(7)

T = -21 N

T = 21 N to the left

6.3 Consider the free body diagrams of the 1 kg or 2 kg blocks; to the right as positive: For 2 kg block:

Fnet = ma

F12 = (2)(7) = 14

 F12 = 14 N

 F12 = 14 N to the right

**OR**

For 1 kg block: Fnet = ma

T + F21 = ma

21 + F21 = (1)(7)

 F21 = -14 N

 F12 = 14 N to the left

**Step 4: Evaluate the answer**

The force exerted by the 1 kg block on the 2 kg is to the right (positive sign according to sign convention) whilst the force exerted by the 2 kg block on the 1 kg block is to the left (negative sign according to sign convention). This phenomenon is in line with Newton's third law of motion.

1. **TWO BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, BOTH ON A FLAT HORIZONTAL PLANE WITH FRICTION.**

Two blocks of masses 2 kg and 3 kg, joined by a light inelastic string, move on a rough horizontal

surface under the influence of a force of 42 N as shown. The coefficients of kinetic friction between the surface and the 2 kg and 3 kg blocks are 0,1 and 0,15 respectively.

2 kg 3 kg

Calculate the tension in the rope joining the two blocks.

**SOLUTION**

**Step 1: Draw a free body diagram** for each block.

2 kg: N

f T

w

3 kg: T

f

N

42 N

w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 Both objects experience the same acceleration in the same direction. Therefore we can consider the system of two blocks when calculating the acceleration (method 1 below) OR we

can consider each object separately (method 2 below).

 Different frictional forces act on the two blocks – therefore the frictional force on each should be calculated separately.

 To find the tension in the rope each block should be isolated and Newton’s second law should

be applied to each block separately.

**Step 3: Solve**

**Method 1**

**Important note:** T on the 2 kg block is equal in magnitude but opposite in direction to T on the

3 kg block. Therefore, when considering the system of two blocks, these two forces will cancel and thus will not form part of the calculation.

Consider the system of two blocks; to the right as positive: Fnet = ma

F + f2 kg + f3 kg = ma (The net force is the vector sum of all the forces acting on the blocks.)

F + k N2 kg + k N3 kg = ma

42 - (0,1)(2)(9,8) - (0,15)(3)(9,8) = (2 + 3)a

a = 7,13 m∙s-2

Consider any one of the blocks to find T: T on 2 kg block:

Fnet = ma

T + f = ma (The net force is the vector sum of all the forces acting on the

block.)

T + k N = ma

T + k mg = ma

T - (0,1)(2)(9,8) = 2(7,13)

T = 16,21 N T = 16,21 N to the right (in the string) T on 3 kg block; to the right as positive:

Fnet = ma

T + f + F = ma (The net force is the vector sum of all the forces acting on the block.)

T + k N + F = ma

T + k mg + F = ma

T - (0,15)(3)(9,8) + 42 = 3(7,13)

T = - 16,21 N

T = 16,21 N to the left (in the string)

**Method 2:**

This method considers each block separately. An equation with two unknowns is obtained for each block. T is obtained by solving these simultaneous equations.

**Important: When using simultaneous equations** to solve for the tension in the string joining two objects, it must be remembered that the force that the string exerts on one object is equal to but opposite in direction to the force that the string exerts on the other object. Therefore, **when deriving the two equations, the tension (T) should be given opposite signs.**

Consider the 2 kg block; to the right as positive: Fnet = ma

T + f = ma (The net force is the vector sum of all the forces acting on the block.)

T + k N = ma

T + k mg = ma

T - (0,1)(2)(9,8) = 2a (T acts to the right and is given a positive sign when substituting.) T – 1,96 = 2a ……………(1)

Consider the 3 kg block; to the right as positive:

Fnet = ma

T + f + F = ma (The net force is the vector sum of all the forces acting on the block.)

T + k N + F = ma

T + k mg + F = ma

- T - (0,15)(3)(9,8) + 42 = 3a (T acts to the left and is given a negative sign when substituting.)

- T + 37,59 = 3a …………(2)

Equation (1) + equation (2):

35,63 = 5a

a = 7,13 m∙s-2

T – 1,96 = 2a …………….(1)

T – 1,96 = 3(7,13)

T = 16,22 N

T = 16,22 N, to the right

**OR**

- T + 37,59 = 3a …………(2)

-T + 37,59 = 3(7,13)

T = 16,2 N

**Step 4: Evaluate the answer**

Both methods give the same answer. It is important to know that whenever the tension in a string joining two objects needs to be calculated, a free body diagram for each object is required. The system method (method 1) can only be used to calculate the acceleration.

1. **TWO-BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, ONE ON A HORIZONTAL PLANE WITHOUT FRICTION, AND A SECOND HANGING VERTICALLY FROM A STRING OVER A FRICTIONLESS PULLEY.**

In the diagram below, a 1 kg mass on a smooth horizontal surface is joined to a 2 kg mass by a light, inextensible string running over a frictionless pulley.

1 kg

2 kg

Calculate the tension in the string.

**SOLUTION**

**Step 1: Draw a free body diagram**

1 kg: N

2 kg: T

T

w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 No normal force is included in the free body diagram of the 2 kg mass. The normal force is a force due to the contact between two surfaces; hence a normal force is a contact force. The

2 kg mass is not resting on any surface – therefore there is no normal force.

 The magnitude of the acceleration for both masses has the same value *a*. The directions of the accelerations are not the same. The 1 kg mass moves horizontally to the right and the 2

kg mass moves vertically downward.

**Step 3: Solve**

**Method 1:**

The acceleration experienced by the two objects is equal in magnitude and can be calculated using the system approach. The system consists of the two masses. The force that the string exerts on the 1 kg mass is equal to and opposite in direction to the force that the string exerts on the 2 kg mass. Therefore the two tension forces cancel and the weight of the 2 kg mass is the net force acting on the system of two masses.

System of mass 3 kg; downward as positive: Fnet = ma

w = ma

(2)(9,8) = (2 + 1)a

a = 6,53 m∙s-2

a = 6,53 m∙s-2, downward

To calculate T, consider the free body diagram of any one of the masses: For 1 kg mass, to the right as positive:

T = ma

 T = (1)(6,53) = 6,53 N

 T = 6,53 N, to the right (the force of the string on the 1 kg mass is to the right)

**OR**

For 2 kg mass, downward as positive: Fnet = ma

T + w = ma

T + (2)(9,8) = 2(6,53)

 T = -6,53 N

 T = 6,53 N, upwards (the string exerts an upwards force on the 2 kg mass)

**Method 2:**

In this method, the two masses are considered separately. From the two free body diagrams, two equations with two unknowns each are obtained. T is obtained by solving these simultaneous equations.

**Important: When using simultaneous equations** to solve for the tension in the string joining two objects, it must be remembered that the force that the string exerts on one object is equal to but opposite in direction to the force that the string exerts on the other object. Therefore, **when deriving the two equations, the tension (T) should be given opposite signs.**

1 kg mass; to the right as positive: Fnet = ma

T = ma (T acts to the right and is given a positive sign when substituting.)

T = (1)a

T = a ……………………………(1)

2 kg mass; downward as positive:

Fnet = ma

T + w = ma

-T + (2)(9,8) = 2a (T acts upwards and is given a negative sign when substituting.)

-T + 19,6 = 2a …………………..(2)

(1) in (2):

-T + 19,6 = 2T

T = 6,53 N

**Step 4: Evaluate the answer**

Both methods give the same answer. Although the system approach appears quick and easy, it does not give any information about the internal forces such as tension. To find the tension, you must consider the free body diagram of one of the masses separately.

1. **TWO-BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, ONE ON A HORIZONTAL PLANE WITH FRICTION, AND A SECOND HANGING VERTICALLY FROM A STRING OVER A FRICTIONLESS PULLEY.**

In the diagram below, a 1 kg mass on a rough horizontal surface is joined to a 2 kg mass by a light, inextensible string running over a frictionless pulley. The coefficient of kinetic friction between the 1 kg mass and the surface is 0,13.

1 kg

2 kg

Calculate the tension in the string.

**SOLUTION**

**Step 1: Draw a free body diagram**

1 kg: N

f T

2 kg: T

w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 No normal force is included in the free body diagram of the 2 kg mass. The normal force is a force due to the contact between two surfaces; hence a normal force is a contact force. The

2 kg mass is not resting on any surface – therefore there is no normal force.

 The magnitude of the acceleration for both masses has the same value *a*. The directions of the accelerations are not the same. The 1 kg mass moves horizontally to the right and the 2 kg mass moves vertically downward.

 Only the 1 kg objects experiences a frictional force. Therefore two forces now act on the 1 kg mass in the horizontal plane. The frictional force is calculated using the formula f = k N.

**Step 3: Solve**

**Method 1:**

The acceleration of the masses can be calculated using the system approach. The system consists of the two masses. The tension in the string on the 1 kg mass is equal to and opposite in direction to the tension on the 2 kg mass. Therefore the two tension forces cancel and the weight of the 2 kg mass is the net force acting on the system of two masses.

System of mass 3 kg, downward as positive: Fnet = ma

w + f = ma

w + k N = ma

mg + k mg = ma

(2)(9,8) – (0,13)(1)(9,8) = (3)a

a = 6,11 m∙s-2

a = 6,11 m∙s-2 downward

To calculate T, consider the free body diagram of any one of the masses: For 1 kg mass; to the right as positive:

T + f = ma

T + k N = ma

T + k mg = ma

T – (0,13)(1)(9,8) = (1)(6,11)

 T = 7,38 N

 T = 7,38 N, to the right

**OR**

For 2 kg mass; downward as positive: Fnet = ma

T + w = ma

T + (2)(9,8) = 2(6,11)

 T = -7,38 N

 T = 7,38 N, upwards in the string

**Method 2:**

In this method, the two masses are considered separately. From the two free body diagrams, two equations with two unknowns each are obtained. T is obtained by solving these simultaneous equations.

**Important: When using simultaneous equations** to solve for the tension in the string joining two objects, it must be remembered that the force that the string exerts on one object is equal to but opposite in direction to the force that the string exerts on the other object. Therefore, **when deriving the two equations, the tension (T) should be given opposite signs.**

1 kg mass; to the right as positive: Fnet = ma

T + f = ma

T + k N = ma

T + k mg = ma

T – (0,13)(1)(9,8) = (1)a (T acts to the right and is given a positive sign when substituting.)

T – 1,27 = a ……………………(1)

2 kg mass; downward as positive:

Fnet = ma

mg + T = ma

(2)(9,8) -T = 2a (T acts upwards and is given a negative sign when substituting.)

19,6 -T = 2a ……………………..(2)

(1) in (2):

19,6 -T = 2(T – 1,27)

T = 7,38 N

**Step 4: Evaluate the answer**

Both methods give the same answer. Although the system approach appears quick and easy, it does not give any information about the internal forces such as tension. To find the tension, you must consider the free body diagram of one of the masses separately.

1. **TWO-BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, BOTH ON AN INCLINED PLANE WITHOUT FRICTION.**

Two objects of mass 6 kg and 3 kg respectively are connected by a light inelastic string. They are

pulled up a frictionless inclined plane which makes an angle of 30° with the horizontal, with a force of magnitude F. The mass of the string may be ignored.

F

3 kg

6 kg

30°

Calculate the:

10.1 Tension in the string if the system accelerates up the inclined plane at 4 m∙s-2

10.2 Magnitude of F if the system moves up the inclined plane at CONSTANT VELOCITY

**SOLUTION**

**Step 1: Draw a free body diagram**

N N

6 kg: 3 kg: T

F

T

w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 In Q10.1, the two objects experience the same acceleration. The acceleration is given and the only way to calculate the tension in the string is to consider the 6 kg object. The 3 kg

object has two unknown forces i.e. F and T, acting on it.

 In Q10.2, the acceleration is zero and the objects move up the incline at constant velocity.

The net force acting on the system is zero. The easiest in this case is to follow the system method. Note that the tension in the string in this case will be different from the tension in

Q10.2 where the acceleration is not zero.

 The tension in the string exerted on the 6 kg object is equal in magnitude, but opposite in direction to the tension exerted on the 3 kg object. Therefore, when substituting, the sign of T

in an equation using the 6 kg object will be opposite to that of T when using the 3 kg object.

**Step 3: Solve**

10.1

The two objects are considered separately. Usually, from the two free body diagrams, two equations with two unknowns each are obtained. In this case, the acceleration is known and therefore only the 6 kg object is considered. No simultaneous equations will thus be used.

6 kg object; upwards along the incline as positive:

Fnet = ma

T + w// = ma

T + mgsin30° = ma (The net force is the vector sum of all the forces acting on the

object.)

T – (6)(9,8)sin30° = 6(4) (Apply chosen sign convention when substituting.) T – 29,4 = 24

T = 53,4 N

T = 53,4 N, upwards along the incline

10.2

**Method 1:**

Whole system; upwards along the incline as positive: Fnet = ma

F + w//(6 kg + 3kg = ma

F + mgsin30° = ma

F – (9)(9,8)sin30° = 0

F = 44,1 N

F = 44,1 N, upwards along the incline

**Method 2:**

Consider the free body diagram of each object separately.

**Important: When using simultaneous equations** to solve for the tension in the string joining two objects, it must be remembered that the force that the string exerts on one object is equal to but opposite in direction to the force that the string exerts on the other object. Therefore, **when deriving the two equations, the tension (T) should be given opposite signs.**

6 kg object; upwards along the incline as positive: Fnet = ma

T + w// = ma

T + mgsin30° = ma (The net force is the vector sum of all the forces acting on the object.)

T – (6)(9,8)sin30° = 0 (T on 6 kg object is upwards along incline, thus positive sign.) T – 29,4 = 0

T = 29,4 N

3 kg object; upwards along the incline as positive: Fnet = ma

F + w// + T = ma

F + mgsin30° + T = ma (The net force is the vector sum of all the forces acting on the

object.)

F – (3)(9,8)sin30° - 29,4 = 0 (T on 3 kg object is downward along incline, thus negative sign.)

F = 44,1 N

F = 44,1 N, upwards along the incline

**Step 4: Evaluate the answer**

The system method provides the answer to Q10.1 faster. With a little more effort, the same answer can be obtained when considering the free body diagram of each object separately. An important observation in this problem is that the tension calculated in Q10.1 cannot be substituted when solving Q10.2. The acceleration in Q10.1 differ from that on Q10.2 and therefore the applied force F as well as the tension in the string will be different.

1. **TWO-BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, BOTH ON AN INCLINED PLANE WITH FRICTION.**

Two objects of mass 6 kg and 3 kg respectively are connected by a light inelastic string. They are

pulled up an inclined plane which makes an angle of 30° with the horizontal, with a force of magnitude F. The mass of the string may be ignored.

F

3 kg

6 kg

30°

The coefficient of kinetic friction for the 3 kg object and 6 kg object respectively is 0,1 and 0,2.

Calculate the:

11.1 Tension in the string if the system accelerates up the inclined plane at 4 m∙s-2

11.2 Magnitude of F if the system moves up the inclined plane at CONSTANT VELOCITY

**SOLUTION**

**Step 1: Draw a free body diagram**

N N

6 kg: 3 kg: T

F

f T

f

w w

**Step 2: Identify the formula**

Points to consider:

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 In Q10.1, the two objects experience the same acceleration. The acceleration is given and the only way to calculate the tension in the string is to consider the 6 kg object. The 3 kg

object has two unknown forces i.e. F and T, acting on it.

 In Q10.2, the acceleration is zero and the objects move up the incline at constant velocity.

The net force acting on the system is zero. The easiest in this case is to follow the system method. Note that the tension in the string in this case will be different from the tension in

Q10.1 where the acceleration is not zero.

 The tension in the string exerted on the 6 kg object is equal in magnitude, but opposite in direction to the tension exerted on the 3 kg object. Therefore, when substituting, the sign of T

in an equation using the 6 kg object will be opposite to that of T when using the 3 kg object.

 The normal force is needed to calculate the frictional force on each object. The magnitude of the normal force is equal to the magnitude of the component of weight perpendicular to the

inclined plane i.e. mgcos30°. The two objects experience different frictional forces and

therefore a frictional force for each, using the normal force exerted on each, should be calculated.

**Step 3: Solve**

11.1

The two objects are considered separately. Usually, from the two free body diagrams, two equations with two unknowns each are obtained. In this case, the acceleration is known and

therefore only the 6 kg object is considered. No simultaneous equations will thus be used.

6 kg object; upwards along the incline as positive:

6 kg object; upwards along the incline as positive: Fnet = ma

T + f + w// = ma (The net force is the vector sum of all the forces acting on the object.)

T + k N + mgsin30° = ma

T + k mgcos30° + mgsin30° = ma

T – (0,2)(6)(9,8)cos30° - (6)(9,8)sin30° = 6(4)

T = 63,58 N

11.2

**Method 1:**

Whole system; upwards along the incline as positive: Fnet = ma

F + f6 kg + f3 kg + w//(6 kg + 3 kg) = ma

F + k N6 kg + k N3 kg + mgsin30°= ma

F + k mgcos30°+ k mgcos30° + mgsin30°= ma

F – (0,2)(6)(9,8)cos30° – (0,1)(3)(9,8)cos30° – (3 + 6)(9,8)sin30° = 0 F = 56,83 N

**Method 2:**

Consider the free body diagram of each object separately. Usually, from the two free body diagrams, two equations with two unknowns each are obtained. In this case, the acceleration is known and therefore only the 6 kg object is considered. No simultaneous equations will thus be used.

6 kg; upwards along the incline as positive: Fnet = ma

T + f + w// = ma

T + k N6 kg + mgsin30° = ma

T + k mgcos30° + mgsin30° = ma

T – (0,2)(6)(9,8)cos30° - (6)(9,8)sin30° = 0 (T on 6 kg object is upwards along incline, thus +

sign.)

T – 39,58 = 0 T = 39,58 N ………………….(1)

3 kg; upwards along the incline as positive: Fnet = ma

F + f + w// + T = ma

F + k N3 kg + mgsin30° + T = ma

F + k mgcos30° + mgsin30° + T = ma

F – (0,1)(3)(9,8)cos30° - (3)(9,8)sin30° - T = 0 (T on 3 kg is downward along incline, thus - sign.) F – T = 17,25 …………………(2)

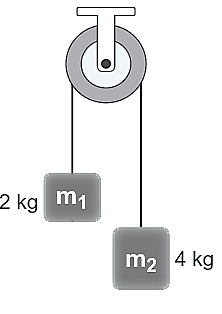
Equation (1) into equation (2): F – 39,58 = 17,25

F = 56,83 N F = 56,83 N, upwards along the incline

**Step 4: Evaluate the answer**

Due to the presence of frictional forces, the tension in Q11.1 is greater than that in Q10.1. Also, due to the presence of frictional forces, the applied force in Q11.2 is greater than that in Q10.2.

1. **TWO-BODIES JOINED BY A LIGHT INEXTENSIBLE STRING, BOTH HANGING VERTICALLY FROM A STRING OVER A FRICTIONLESS PULLEY.**



Two blocks, one with a mass of 2 kg and the other with a mass of 4 kg, hang over a frictionless pulley on a thin, light rope.

Calculate the:

12.1 Acceleration of the blocks

12.2 Tension in the rope

**SOLUTION**

**Step 1: Draw a free body diagram**

2 kg: T

4 kg: T

w1 = m1g

**Step 2: Identify the formula**

Points to consider:

w2 = m2g

 The key equation for any problem that relates forces and motion is Newton's Second Law.

Regardless of what quantity you are asked to find, begin with the Second Law i.e. Fnet = ma.

 Because the pulley turns easily (is frictionless), the tension in the rope is the same on both sides. Because the rope does not stretch, the magnitude of the acceleration will be the same

for both blocks.

 The system of two masses will accelerate in the direction of the larger mass i.e. the 4 kg block. Therefore the 2 kg block will accelerate upwards and the 4 kg block will accelerate

downward.

 As an intuitive approach to this problem, the total mass (4 kg + 2 kg) is accelerated by a force equal to the difference in hanging weights. This method can only be used to calculate the acceleration of the system. To calculate the tension in the rope, the two objects need to be considered separately.

 In problems like this one, it is convenient to consider the direction of motion as positive. The tension in the rope on the two objects will be equal in magnitude but opposite in direction.

**Step 3: Solve**

12.1

**Method 1:**

The acceleration is numerically the same for the two objects so they can be treated as a system with mass m1 + m2 = 6 kg. The net force on the system is equal to the difference of the weights.

12.1

Downward positive: Fnet = ma

m2g – m1g = (m1 + m2)a

(4)(9,8) – (2)(9,8) = (4 + 2)a

 a = 3,27 m∙s-2

 a = 3,27 m∙s-2 downward

12.2

To find the tension in the rope, the free body diagram of at least one of the objects needs to be considered.

2 kg object; upwards (direction of motion) positive:

Fnet = ma

T + w1 = m1a

T + m1g = m1a

T - (2)(9,8) = 2a

T - 19,6 = 2(3,27)

T = 26,14 N

T = 26,14 N, upwards

4 kg object; downward (direction of motion) positive: Fnet = ma

w2 + T = m2a

m2g + T = m2a

(4)(9,8) + T = 4a

39,2 + T = 4(3,27) T = -26,14

T = 26,14 N upwards

**Method 2:**

12.1

Consider the free body diagram of each object separately.

**Important: When using simultaneous equations** to solve for the tension in the string joining two objects, it must be remembered that the force that the string exerts on one object is equal to but opposite in direction to the force that the string exerts on the other object. Therefore, **when deriving the two equations, the tension (T) should be given opposite signs.**

2 kg object; upwards (direction of motion) positive: Fnet = ma

T + w1 = m1a

T - m1g = m1a

T - (2)(9,8) = 2a (T on 2 kg object is upwards, thus positive sign.) T - 19,6 = 2a

T = 19,6 + 2a ………………..(1)

For the 4 kg object, downward (direction of motion) positive: Fnet = ma

w2 + T = m2a

m2g + T = m2a

(4)(9,8) - T = 4a (T on 4 kg object is upwards, thus negative sign.)

39,2 - T = 4a

-T = -39,2 + 4a

T = 39,2 - 4a ……………….(2)

Equation (1) into equation (2):

19,6 + 2a = 39,2 - 4a  a = 3,27 m∙s-2

The 2 kg block accelerates at 3,27 m∙s-2 upwards

The 4 kg block accelerates at 3,27 m∙s-2 downward.

12.2

From equation (1) for 2 kg object: T = 19,6 + 2a

= 19,6 + 2(3,27) = 26,14 N upwards

**OR**

From equation (2) for 4 kg object:

T = 39,2 - 4a = 39,2 - 4(3,27) = 26,14 N upwards

**Step 4: Evaluate the answer**

Both methods give the same answer. When the tension is needed, it will be better to use method 1 from the start. When only the acceleration is needed, method 2 will be the shorter method. Due to rounding, the final answers obtained might differ slightly.