EUCLIDEAN GEOMETRY PROOFS GR 11 THEOREMS

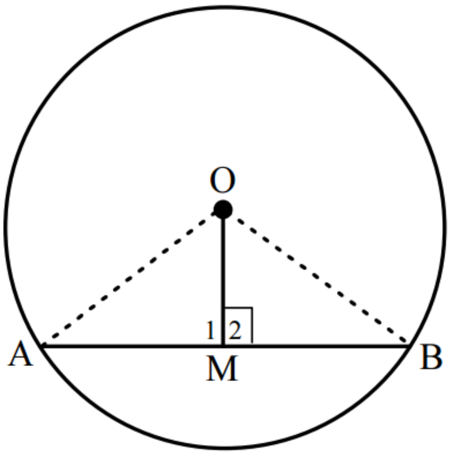
THEOREM 1(A)

The perpendicular drawn from the centre of a circle to a chord bisects the chord.

Theorem 1 in a nutshell according to the given circle sketched below: If $OM \perp AB$ (which means that $\hat{M}_1 = \hat{M}_2 = 90°$) then $AM = MB$

Given: Circle with centre O with $OM \perp AB$. AB is a chord

Required to prove: $AM = MB$.



Proof

Join OA and OB.

In ΔOAM and ΔOBM :

(a) $OA = OB$ ….. radii

(b) $\hat{M}_1 = \hat{M}_2 = 90°$ ….. given

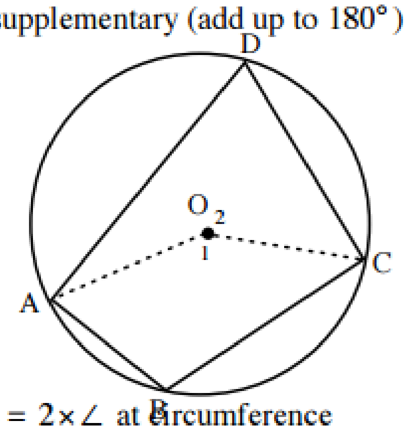
(c) $OM = OM$ ….. common

$\therefore \Delta OAM \equiv \Delta OBM$ ….. RHS

$\therefore AM = MB$

THEOREM 5

The opposite angles of a cyclic quadrilateral are supplementary (add up to $180°$)
(opp $\angle s$ cyclic quad)



Given: A, B, C and D are points that lie on the circumference of the circle (ABCD is a cyclic quadrilateral)

Required to prove: $\hat{A} + \hat{C} = 180°$ and $\hat{B} + \hat{D} = 180°$

Proof

Join AO and OC.

$\hat{O}_1 = 2\hat{D}$ ….. $\angle$ at centre $= 2 \times \angle$ at circumference

$\hat{O}_2 = 2\hat{B}$ ….. $\angle$ at centre $= 2 \times \angle$ at circumference

$\hat{O}_1 + \hat{O}_2 = 2\hat{D} + 2\hat{B}$

And $\hat{O}_1 + \hat{O}_2 = 360°$ ….. $\angle 's$ at a point

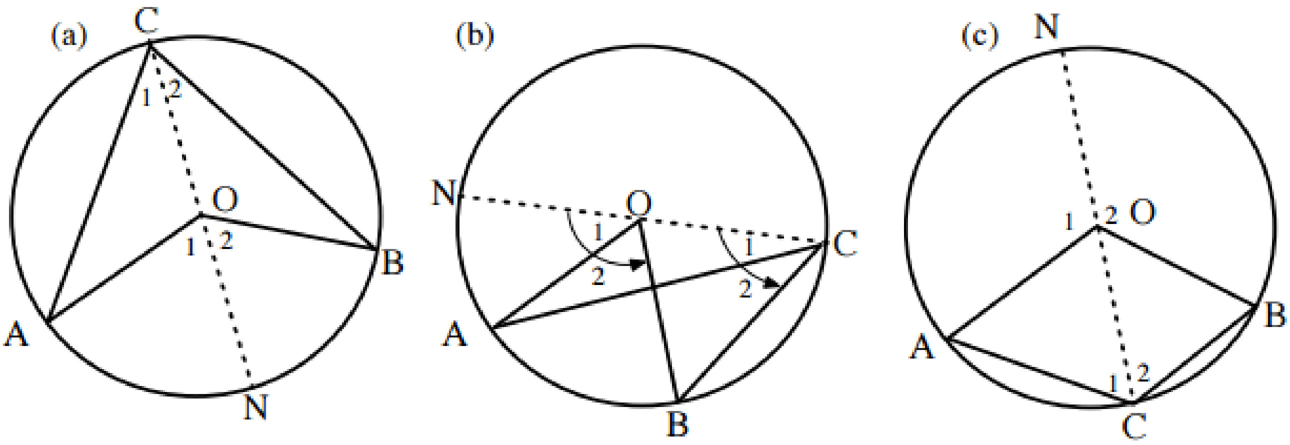
$\therefore 360° = 2\left(\hat{D} + \hat{B}\right)$

$\therefore 180° = \hat{D} + \hat{B}$

Similarly, by joining BO and DO, it can be proven that $\hat{A} + \hat{C} = 180°$

THEOREM 2

The angle which an arc of a circle subtends at the centre of a circle is twice the angle it subtends at the circumference of the circle. (∠ at the centre = 2×∠ at circumference)



Given: Circle with centre O and A, B and C are all points on the circumference of the circle.

Required to prove: $A\hat{O}B = 2A\hat{C}B$

For diagrams (a) and (c)

Proof:

Join CO and produce to N.

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ ..... Ext ∠ of ΔOAC

But $\hat{C}_1 = \hat{A}$ ..... OA = OC , Radii

$\therefore \hat{O}_1 = 2\hat{C}_1$

Similarly, in ΔOCB $\hat{O}_2 = 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$

$\therefore \hat{O}_1 + \hat{O}_2 = 2(\hat{C}_1 + \hat{C}_2)$

$\therefore A\hat{O}B = 2A\hat{C}B$

For diagram (b)

Proof:

Join CO and produce to N.

$\hat{O}_1 = \hat{C}_1 + \hat{A}$ ..... Ext ∠ of ΔOAC

But $\hat{C}_1 = \hat{A}$ ..... OA = OC , Radii

$\therefore \hat{O}_1 = 2\hat{C}_1$

Similarly, in ΔOCB $\hat{O}_2 = 2\hat{C}_2$

$\therefore \hat{O}_2 - \hat{O}_1 = 2\hat{C}_2 - 2\hat{C}_1$

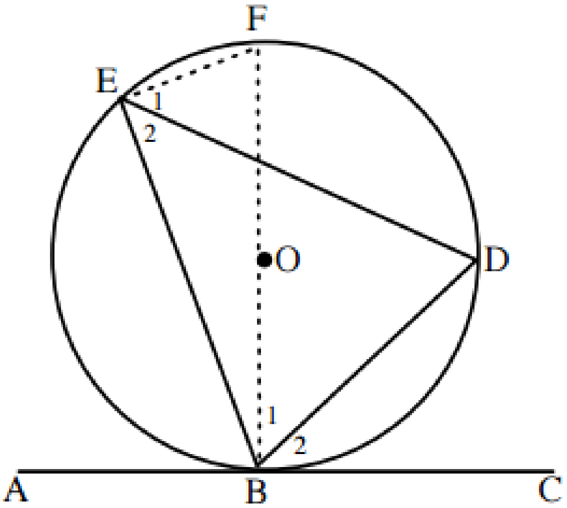
$\therefore \hat{O}_2 - \hat{O}_1 = 2(\hat{C}_2 - \hat{C}_1)$

$\therefore A\hat{O}B = 2A\hat{C}B$

THEOREM 9

The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (Tan-chord)

Acute-angle case



Given: Tangent ABC

Required to prove: $C\hat{B}D = B\hat{E}D$

Proof:

Draw diameter BOF and join EF

$\hat{B}_1 + \hat{B}_2 = 90°$ ….. tan ⊥ rad

$\hat{E}_1 + \hat{E}_2 = 90°$ ….. ∠ in semi-circle

But $\hat{B}_1 = \hat{E}_1$ ….. FD subt = ∠s

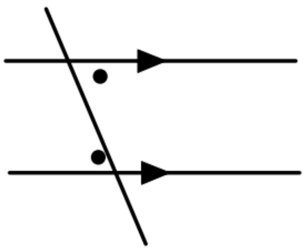
$\therefore \hat{B}_2 = \hat{E}_2$

$\therefore C\hat{B}D = B\hat{E}D$

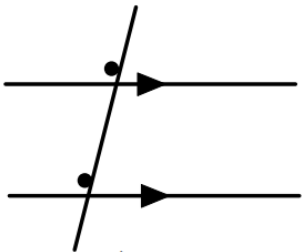
KEY WORDS TO LOOK FOR:

1. Parallel lines: Look for corresponding,alternate and co-interior angles.

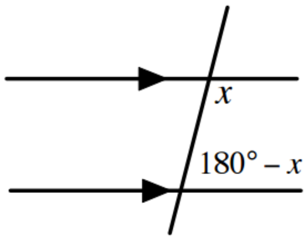
1. When parallel lines are given



alt ∠s =

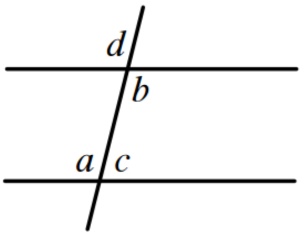


corr ∠s =



co-int ∠s suppl

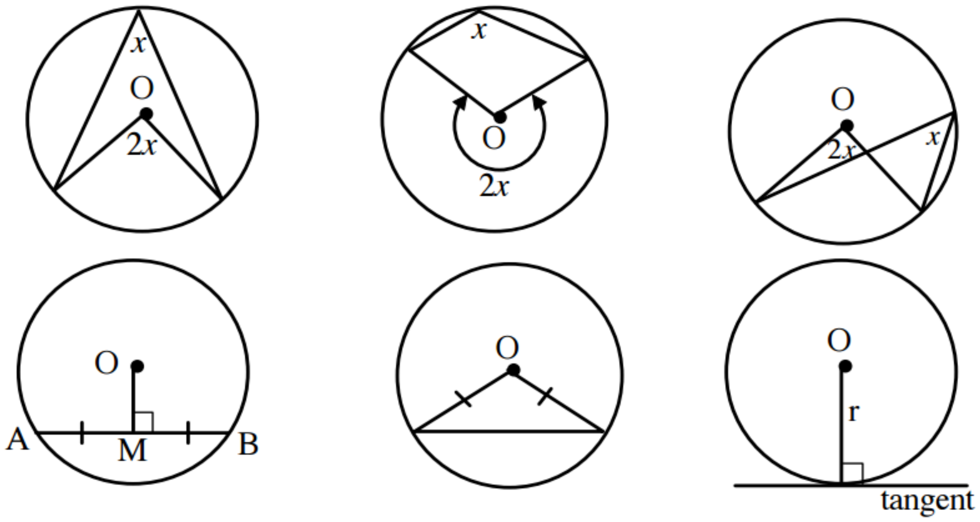
2. How to prove that lines are parallel



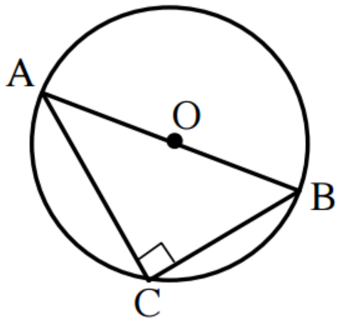
Prove that $a = b$ or $a = d$ or $b + c = 180°$

2 O is the centre of the circle. Look at the question to see which of the following questions you can apply.

Centre of a circle given



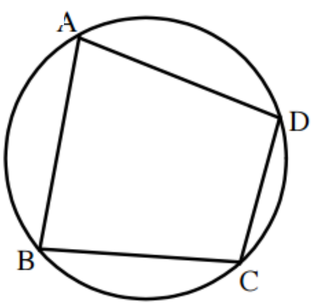
Diameter given



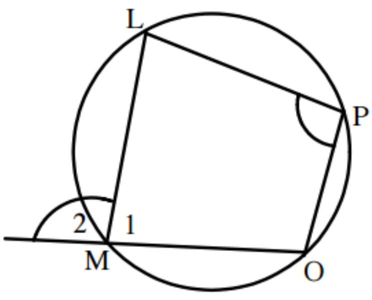
If AOB is diameter then $\hat{C} = 90°$
($\angle$ in semi circle)

3. Cyclic Quadrilaterals: Look at the question to see which of the following questions you can apply.

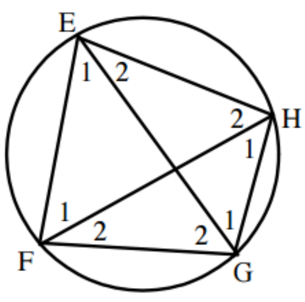
Cyclic quadrilateral given



If ABCD is cyclic then
$\hat{A} + \hat{C} = 180°$ and
$\hat{B} + \hat{D} = 180°$
(opp $\angle s$ of cyclic quad)



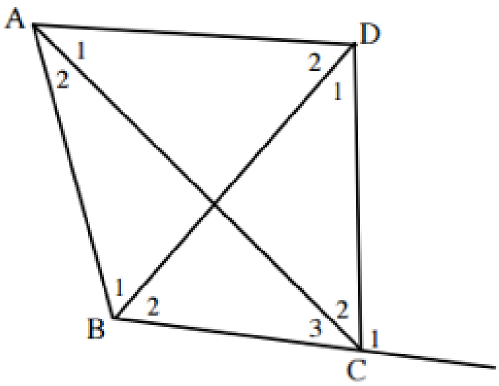
If LMOP is cyclic
then $\hat{M}_2 = \hat{P}$
(Ext $\angle$ cyclic quad)



If EFGH is cyclic then
$\hat{E}_1 = \hat{H}_1$, $\hat{E}_2 = \hat{F}_2$,
$\hat{H}_2 = \hat{G}_2$, $\hat{G}_1 = \hat{F}_1$
($\angle s$ in same segment)

11. How to prove that a quadrilateral is cyclic

ABCD would be a cyclic quadrilateral if you could prove **one** of the following:



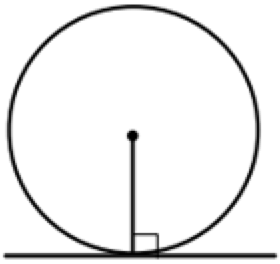
Condition 1:
$\left(\hat{A}_1 + \hat{A}_2\right) + \left(\hat{C}_2 + C_3\right) = 180°$ or
$\left(\hat{B}_1 + \hat{B}_2\right) + \left(\hat{D}_1 + \hat{D}_2\right) = 180°$

Condition 2:
$\hat{C}_1 = \hat{A}_1 + \hat{A}_2$

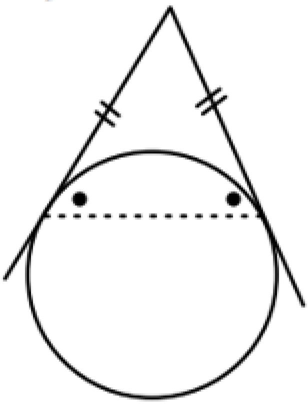
Condition 3:
$\hat{A}_1 = \hat{B}_2$ or $\hat{A}_2 = \hat{D}_1$ or $\hat{B}_1 = \hat{C}_2$ or
$\hat{D}_2 = \hat{C}_3$

4. Tangents: Look at the question to see which of the following questions you can apply

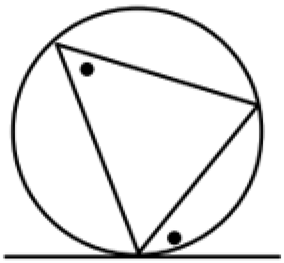
Tangents to circles given

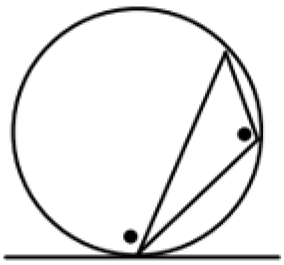


tan ⊥ rad



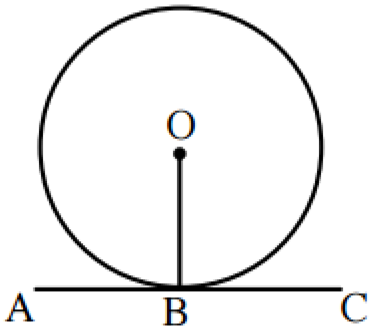
Tangents from the same point



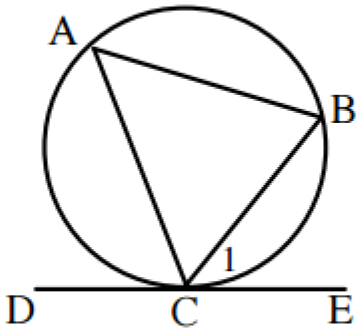


tan chord theorem

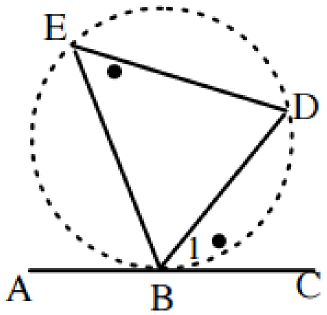
How to prove that a line is a tangent to a circle



ABC is a tangent if $O\hat{B}C = 90°$



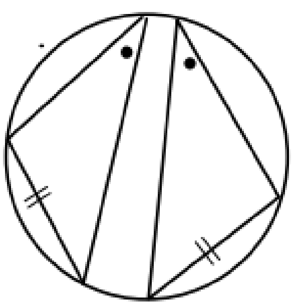
DCE is a tangent if $\hat{C}_1 = \hat{A}$

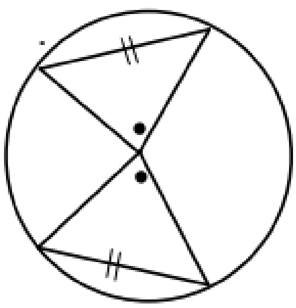


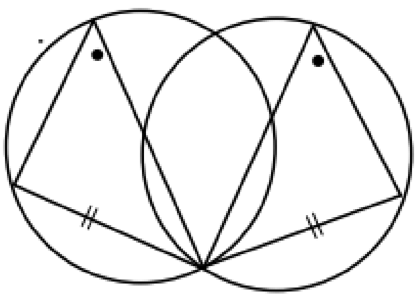
ABC would be a tangent to the "imaginary" circle drawn through EBD if $\hat{B}_1 = \hat{E}$

COROLLARIES ON THEOREM 4

1. Equal chords subtend equal angles at the circumference
2. Equal chords subtend equal angles at the centre
3. Equal chords of equal circles subtend equal angles at the circumference.

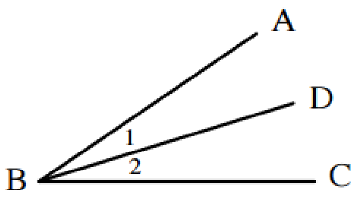




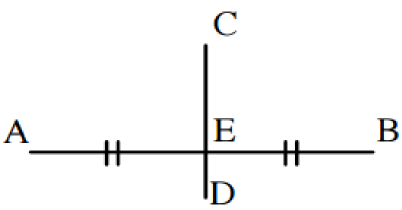


Angle or line bisectors

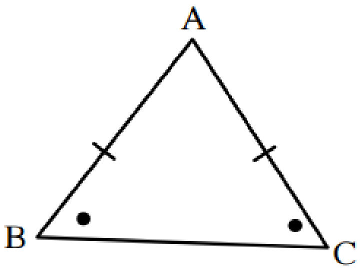
If BD bisects $A\hat{B}C$ then $\hat{B}_1 = \hat{B}_2$



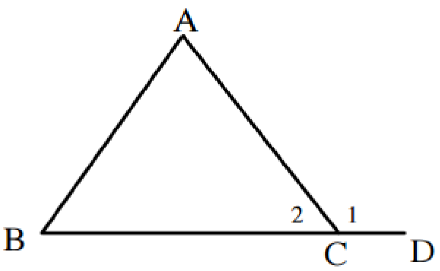
If CD bisects AB then AE = EB



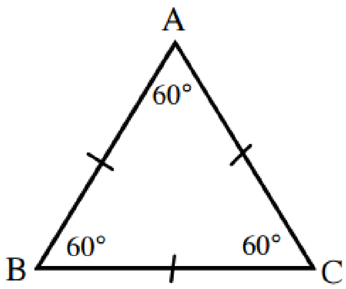
Triangle information



If $\hat{B} = \hat{C}$, then AB = AC.
If AB = AC, then $\hat{B} = \hat{C}$.
ΔABC is **isosceles**

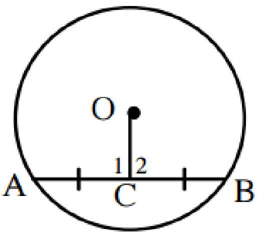


$\hat{A} + \hat{B} + \hat{C}_2 = 180°$ (sum $\angle s$ of Δ)
$\hat{C}_1 = \hat{A} + \hat{B}$ (Ext $\angle$ of Δ)

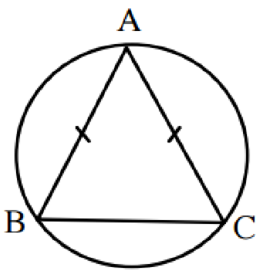


If AB = AC = BC, then
$\hat{A} = \hat{B} = \hat{C} = 60°$
ΔABC is **equilateral**

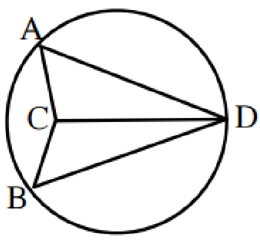
When you must prove two sides are equal



or



or



To prove AC = CB,
prove $\hat{C}_1 = 90°$

To prove AB = AC,
prove $\hat{B} = \hat{C}$

To prove AD = BD,
try prove $\Delta ACD \equiv \Delta BCD$