NOTES FUNCTIONS AND INVERSES 02/03/2024

EXAMPLE 2

Determine the equation of the inverse of $y = 3x^2$.

Solution

Original: $y = 3x^2$

Inverse: $x = 3y^2$

$\therefore y^2 = \frac{x}{3}$

$\therefore y = \pm\sqrt{\frac{x}{3}}$

Note: This inverse gives two outputs for each input. This means that the inverse is **not a function**. More about this later.

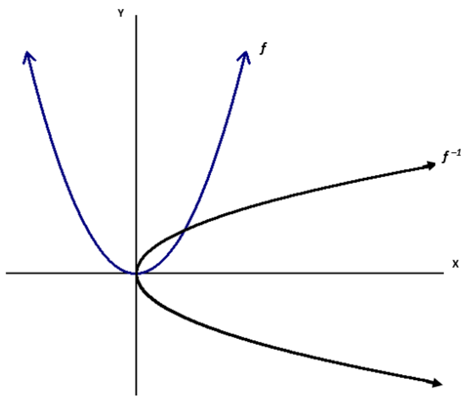
b) Sketch both $f(x) = 3x^2$ and its inverse on the same set of axes.

$f(x) = 3x^2$

x	–1	0	1
y	3	0	3

$f(x)^{-1} = \sqrt{\frac{x}{3}}$

x	3	0	3
y	–1	0	1



EXAMPLE 7

Given the function $y=-2x^2$.

(a) Determine the equation of the inverse of this function.

(b) Sketch the graphs of $y=-2x^2$ and its inverse on the same set of axes and show the line of symmetry.

(c) Determine the coordinates of the points of intersection between $y=-2x^2$ and its inverse.

Solution

(a) Original: $y=-2x^2$

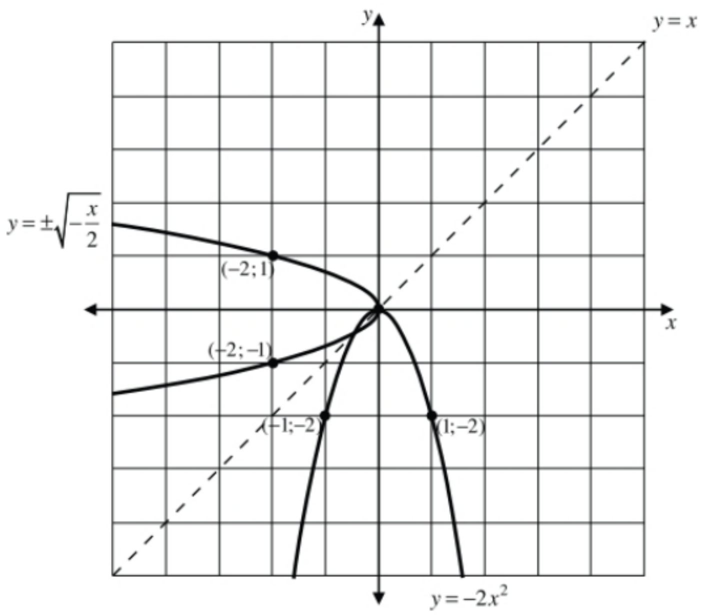
Inverse: $x=-2y^2$

$$\therefore y^2=\frac{x}{-2}$$

$$\therefore y=\pm\sqrt{-\frac{x}{2}}$$

(b) $y=-2x^2$ is a parabola with a negative orientation.

Three points on $y=-2x^2$:	**Invert** coordinates for $y=\pm\sqrt{-\frac{x}{2}}$:
$(-1;-2)$	$(-2;-1)$
$(0;0)$	$(0;0)$
$(1;-2)$	$(-2;1)$



(c) Solve $y=-2x^2$ and $y=x$ simultaneously:

$-2x^2 = x$

$\therefore 2x^2 + x = 0$

$\therefore x(2x+1) = 0$

$\therefore x = 0$ or $x = -\frac{1}{2}$

Points of intersection: $(0\,;0)$ and $\left(-\frac{1}{2}\,;-\frac{1}{2}\right)$

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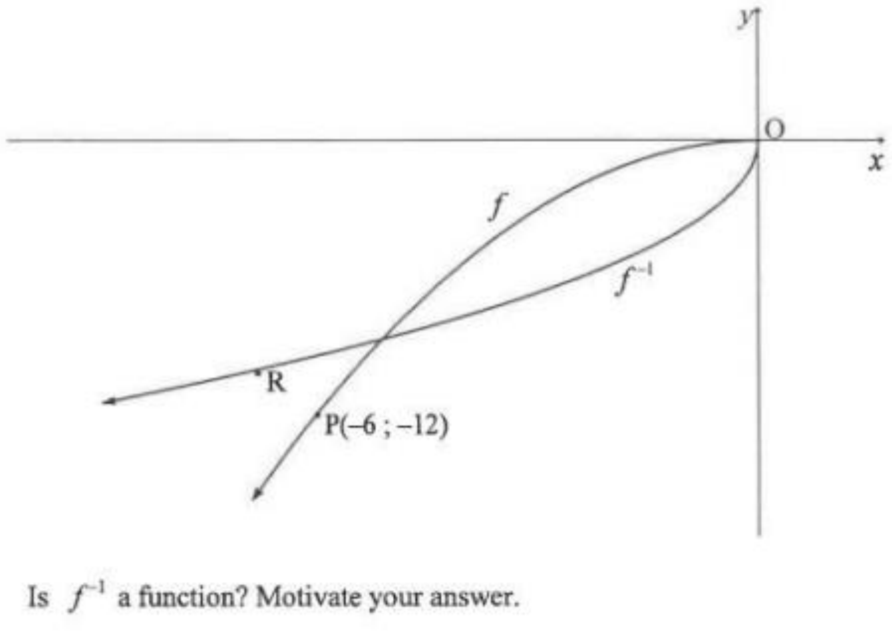
QUESTION 4

In the diagram below, the graph of $f(x)=ax^2$ is drawn in the interval $x \leq 0$.
The graph of f^{-1} is also drawn. P(–6 ; –12) is a point on f and R is a point on f^{-1}.



4.1	Is f^{-1} a function? Motivate your answer.	(2)
4.2	If R is the reflection of P in the line $y=x$, write down the coordinates of R.	(1)
4.3	Calculate the value of a.	(2)
4.4	Write down the equation of f^{-1} in the form $y = ...$	(3) **[8]**

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QUESTION/*VRAAG* 4

4.1	Yes For every x-value there is only one corresponding y value **OR/OF** One to one mapping (vertical line test)	✓answer ✓reason (2)
4.2	R(–12 ; –6)	✓answer (1)
4.3	$f(x)=ax^2$ substitute $(-6;-12)$ $-12=a(-6)^2$ $a=\frac{-1}{3}$	✓substitution ✓answer (2)
4.4	$f: y=-\left(\frac{1}{3}\right)x^2$ $f^{-1}: x=-\left(\frac{1}{3}\right)y^2$ $y^2=-3x$ $y=\pm\sqrt{-3x}$ Only $y=-\sqrt{-3x}$ and $x\leq 0$	✓swapping x and y ✓ $y^2=-3x$ ✓ $y=-\sqrt{-3x}$ (3)
		[8]

6.2 Given: $h(x)=-\sqrt{\frac{x}{3}}$; $x\geq 0$

6.2.1 If $k(x)$ is the inverse of h, give the equation of $k(x)$ (2)

6.2.2 Give the coordinates of the point of intersection of $h(x)$ and $k(x)$ (2)

SOLUTION

6.2.1	$k(x)=3x^2$; $x\leq 0$	✓ $k(x)=3x^2$ ✓ $x\leq 0$	(2)
6.2.2	(0; 0) OR/*OF* origin/ *oorsprong*	✓✓ Answer/ *Antw*	(2)