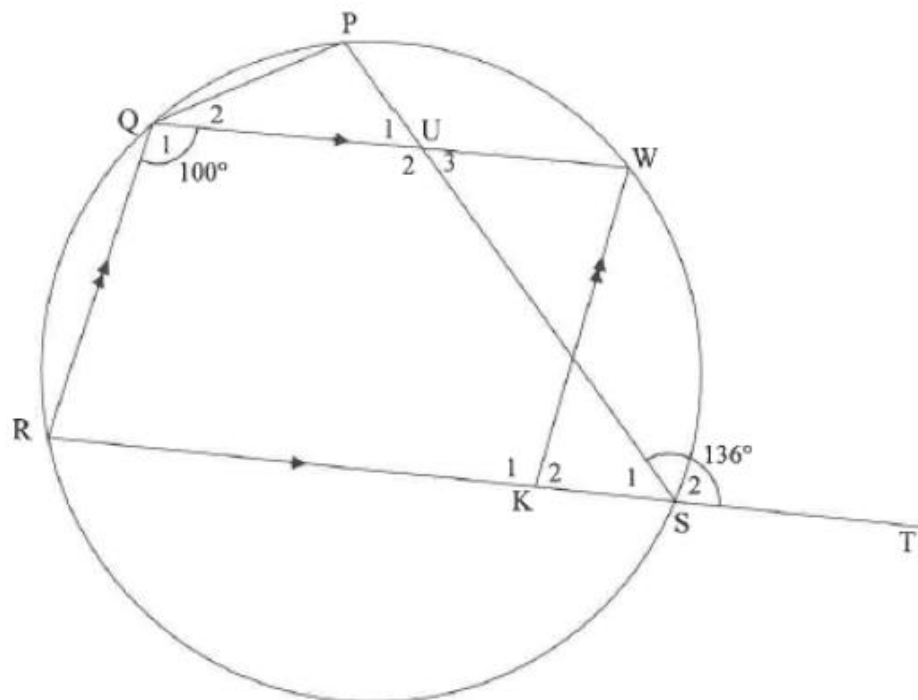


# Euclidean Geometry

November 2019

## QUESTION 8

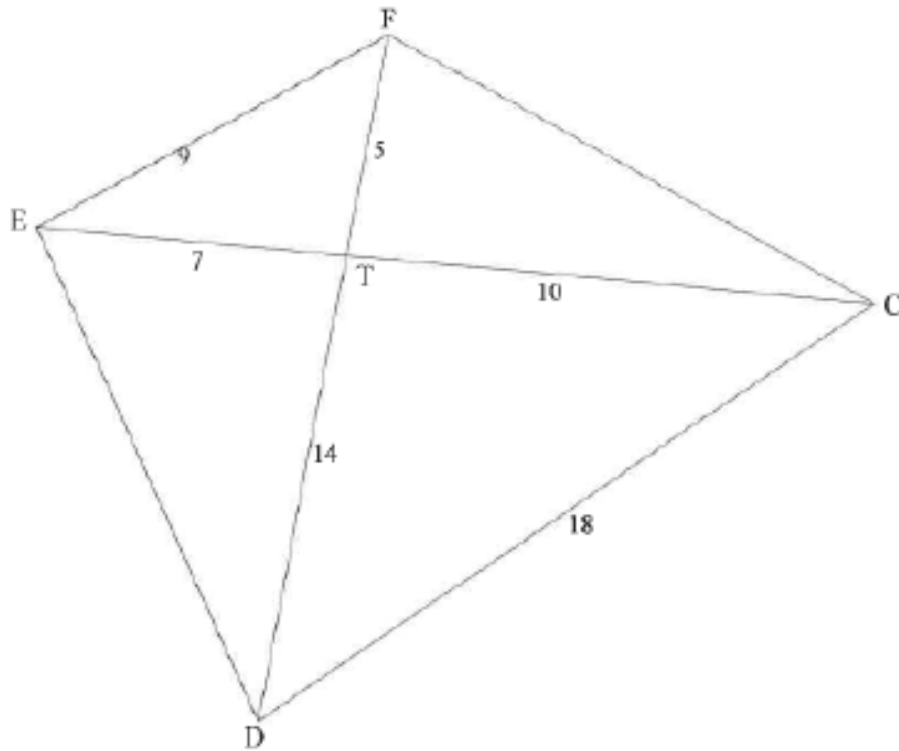
- 8.1 In the diagram, PQRS is a cyclic quadrilateral. Chord RS is produced to T. K is a point on RS and W is a point on the circle such that QRKW is a parallelogram. PS and QW intersect at U.  $\hat{PST} = 136^\circ$  and  $\hat{Q}_1 = 100^\circ$ .



Determine, with reasons, the size of:

- |       |             |     |
|-------|-------------|-----|
| 8.1.1 | $\hat{R}$   | (2) |
| 8.1.2 | $\hat{P}$   | (2) |
| 8.1.3 | $\hat{PQW}$ | (3) |
| 8.1.4 | $\hat{U}_2$ | (2) |

- 8.2 In the diagram, the diagonals of quadrilateral CDEF intersect at T.  
 EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and  
 TD = 14 units.



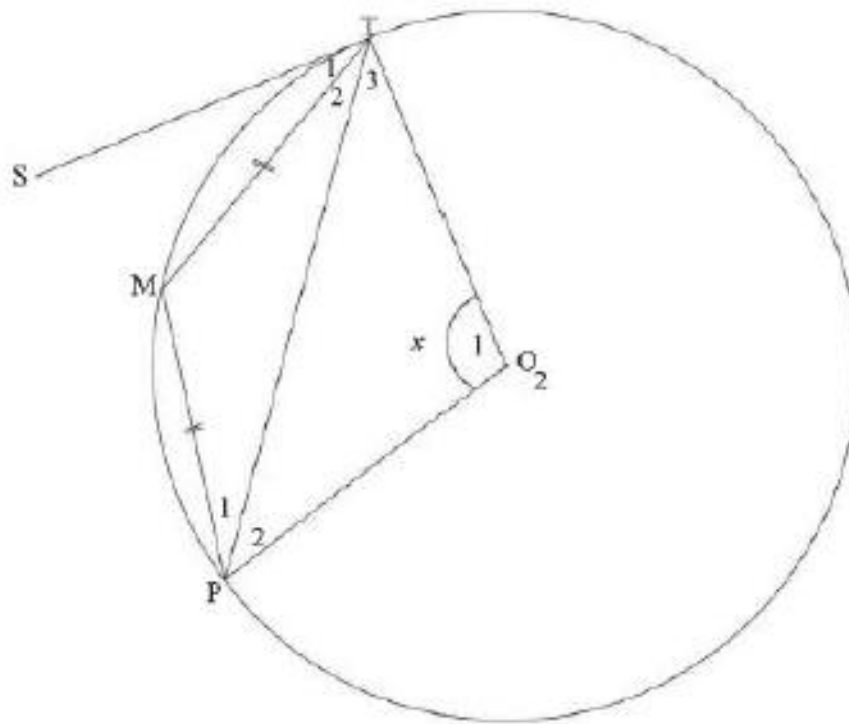
Prove, with reasons, that:

8.2.1  $\hat{EFD} = \hat{ECD}$  (4)

8.2.2  $\hat{DFC} = \hat{DEC}$  (3)

**QUESTION 9**

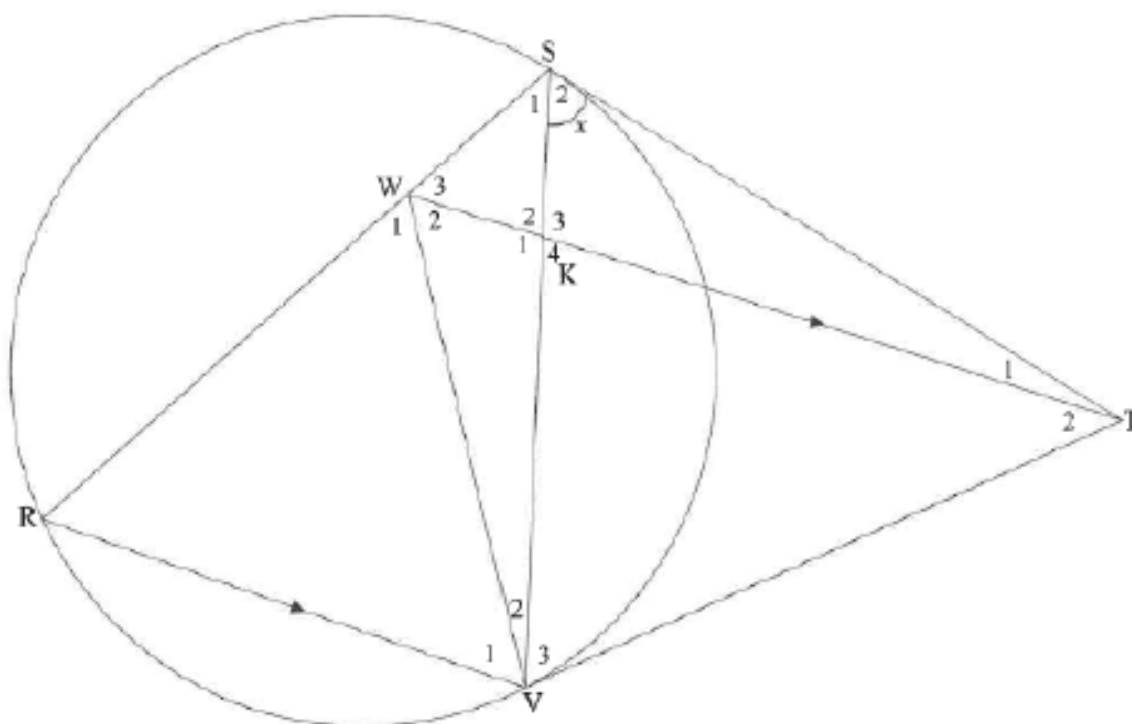
In the diagram,  $O$  is the centre of the circle.  $ST$  is a tangent to the circle at  $T$ .  $M$  and  $P$  are points on the circle such that  $TM = MP$ .  $OT$ ,  $OP$  and  $TP$  are drawn. Let  $\hat{O}_1 = x$ .



Prove, with reasons, that  $\hat{SMT} = \frac{1}{4}x$ .

[7]

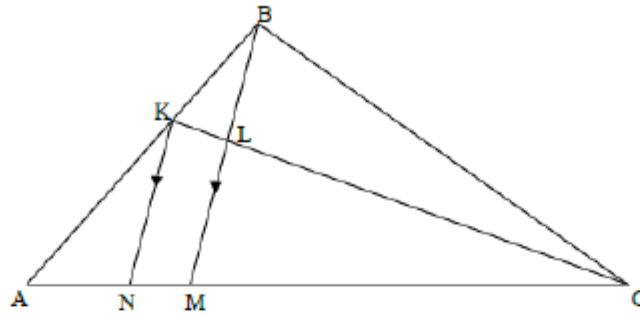
- 10.2 In the diagram,  $ST$  and  $VT$  are tangents to the circle at  $S$  and  $V$  respectively.  $R$  is a point on the circle and  $W$  is a point on chord  $RS$  such that  $WT$  is parallel to  $RV$ .  $SV$  and  $WV$  are drawn.  $WT$  intersects  $SV$  at  $K$ . Let  $\hat{S}_2 = x$ .



- 10.2.1 Write down, with reasons, THREE other angles EACH equal to  $x$ . (6)
- 10.2.2 Prove, with reasons, that:
- (a)  $WSTV$  is a cyclic quadrilateral (2)
  - (b)  $\triangle WRV$  is isosceles (4)
  - (c)  $\triangle WRV \parallel \triangle TSV$  (3)
  - (d)  $\frac{RV}{SR} = \frac{KV}{TS}$  (4)

[25]

- 8.2 In  $\triangle ABC$  in the diagram,  $K$  is a point on  $AB$  such that  $AK : KB = 3 : 2$ .  $N$  and  $M$  are points on  $AC$  such that  $KN \parallel BM$ .  $BM$  intersects  $KC$  at  $L$ .  $AM : MC = 10 : 23$ .



Determine, with reasons, the ratio of:

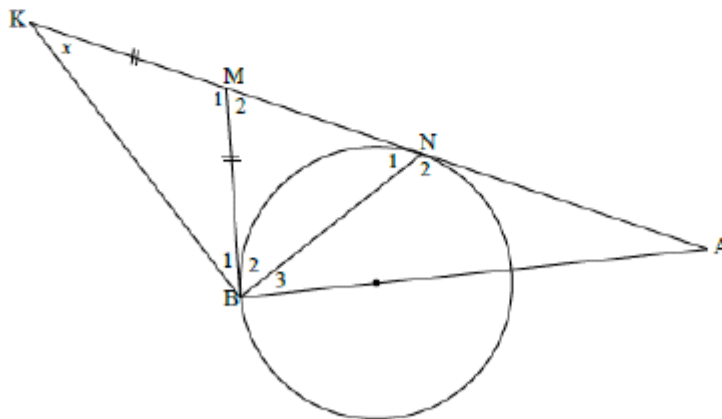
8.2.1  $\frac{AN}{AM}$  (2)

8.2.2  $\frac{CL}{LK}$  (3)

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QUESTION 9

In the diagram, tangents are drawn from point  $M$  outside the circle, to touch the circle at  $B$  and  $N$ . The straight line from  $B$  passing through the centre of the circle meets  $MN$  produced in  $A$ .  $NM$  is produced to  $K$  such that  $BM = MK$ .  $BK$  and  $BN$  are drawn. Let  $\hat{K} = x$ .

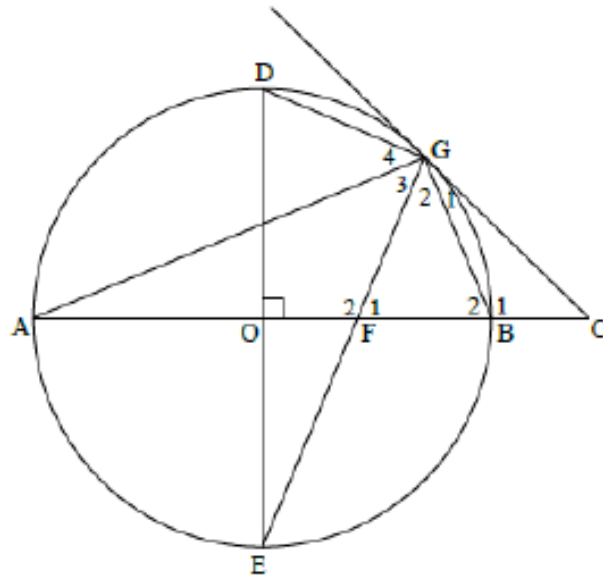


- 9.1 Determine, with reasons, the size of  $\hat{N}_1$  in terms of  $x$ . (6)

- 9.2 Prove that  $BA$  is a tangent to the circle passing through  $K, B$  and  $N$ . (5)

[11]

- 10.2 In the diagram,  $O$  is the centre of the circle and  $CG$  is a tangent to the circle at  $G$ . The straight line from  $C$  passing through  $O$  cuts the circle at  $A$  and  $B$ . Diameter  $DOE$  is perpendicular to  $CA$ .  $GE$  and  $CA$  intersect at  $F$ . Chords  $DG$ ,  $BG$  and  $AG$  are drawn.

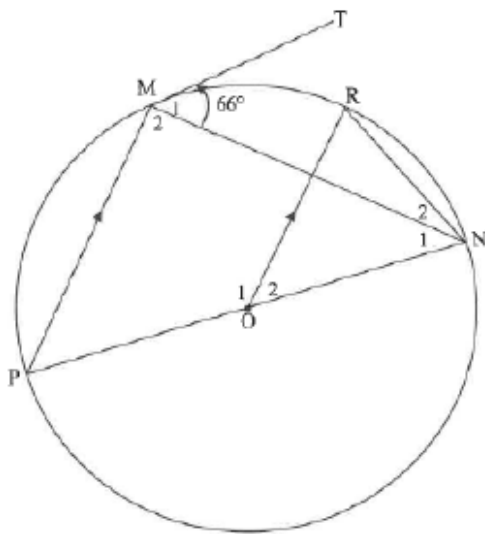


- 10.2.1 Prove that:
- $DGFO$  is a cyclic quadrilateral (3)
  - $GC = CF$  (5)
- 10.2.2 If it is further given that  $CO = 11$  units and  $DE = 14$  units, calculate:
- The length of  $BC$  (3)
  - The length of  $CG$  (5)
  - The size of  $\hat{E}$  (4)

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**QUESTION 8**

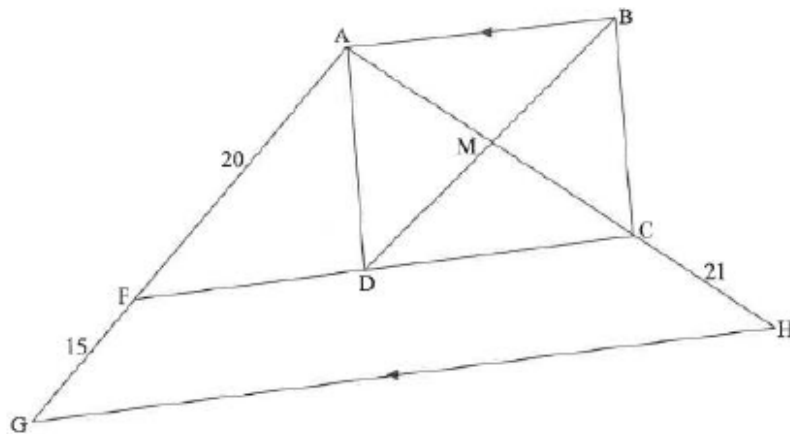
- 8.1  $PON$  is a diameter of the circle centred at  $O$ .  $TM$  is a tangent to the circle at  $M$ , a point on the circle.  $R$  is another point on the circle such that  $OR \parallel PM$ .  $NR$  and  $MN$  are drawn. Let  $\hat{M}_1 = 66^\circ$ .



Calculate, with reasons, the size of EACH of the following angles:

- 8.1.1  $\hat{P}$  (2)
- 8.1.2  $\hat{M}_2$  (2)
- 8.1.3  $\hat{N}_1$  (1)
- 8.1.4  $\hat{O}_2$  (2)
- 8.1.5  $\hat{N}_2$  (3)

8.2 In the diagram,  $\triangle AGH$  is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.

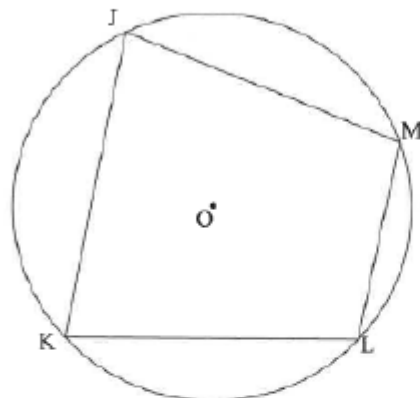


8.2.1 Explain why  $FC \parallel GH$ . (1)

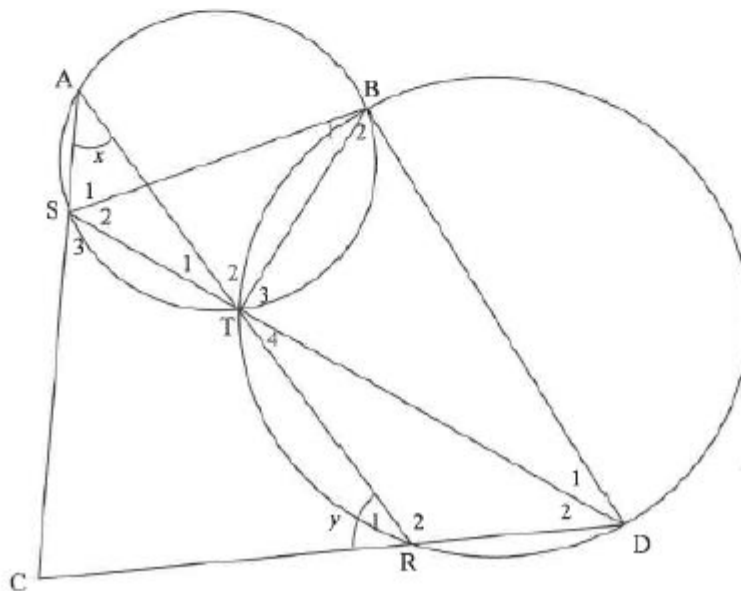
8.2.2 Calculate, with reasons, the length of  $DM$ . (5)  
[16]

**QUESTION 9**

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O. Prove the theorem which states that  $\hat{J} + \hat{L} = 180^\circ$ . (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord. Straight lines STD and ATR are drawn. Chords AS and DR are produced to meet in C, a point outside the two circles.  $\hat{A} = x$  and  $\hat{R} = y$ .

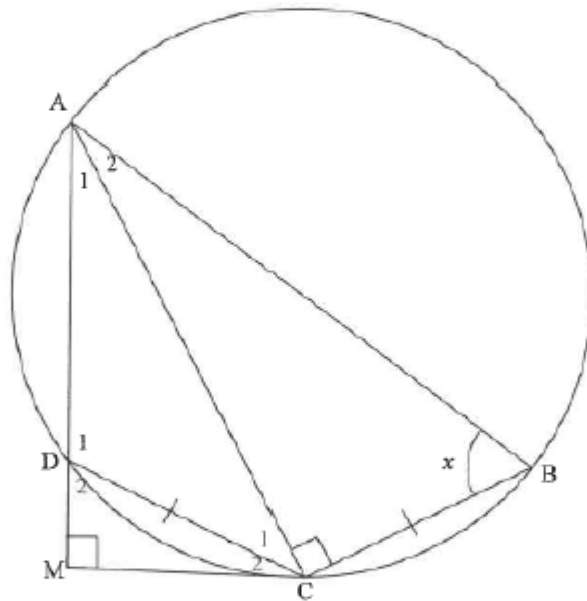




- 9.2.1 Name, giving a reason, another angle equal to:
- (a)  $x$  (2)
- (b)  $y$  (2)
- 9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)
- 9.2.3 It is further given that  $\hat{D}_2 = 30^\circ$  and  $\hat{A}ST = 100^\circ$ .  
 Prove that SD is not a diameter of circle BDS. (4)
- [16]

**QUESTION 10**

In the diagram, ABCD is a cyclic quadrilateral such that  $AC \perp CB$  and  $DC = CB$ . AD is produced to M such that  $AM \perp MC$ . Let  $\hat{B} = x$ .

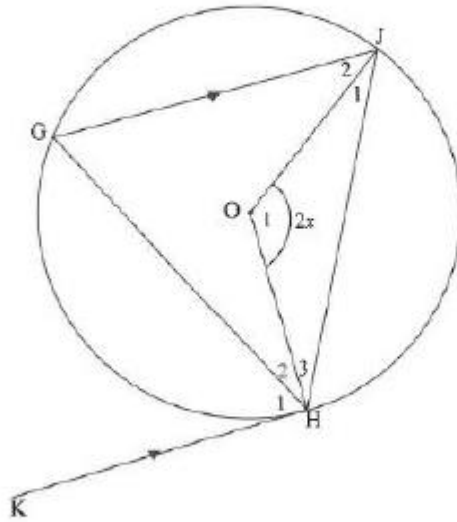


- 10.1 Prove that:
- 10.1.1 MC is a tangent to the circle at C (5)
- 10.1.2  $\triangle ACB \parallel \triangle CMD$  (3)
- 10.2 Hence, or otherwise, prove that:
- 10.2.1  $\frac{CM^2}{DC^2} = \frac{AM}{AB}$  (6)
- 10.2.2  $\frac{AM}{AB} = \sin^2 x$  (2)
- [16]

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QUESTION 8

- 8.1 In the diagram,  $O$  is the centre of the circle. Radii  $OH$  and  $OJ$  are drawn. A tangent is drawn from  $K$  to touch the circle at  $H$ .  $\triangle HGJ$  is drawn such that  $GJ \parallel KH$ .  $\hat{O}_1 = 2x$ .



- 8.1.1 Name, giving reasons, THREE angles, each equal to  $x$ . (5)
- 8.1.2 Prove that  $\hat{H}_2 = \hat{H}_3$ . (3)