SESSION 5

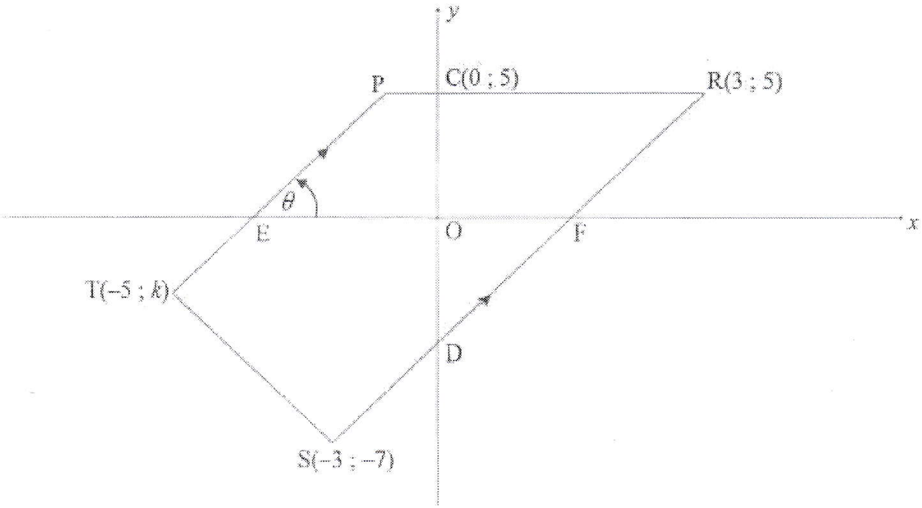
Coordinate Geometry Euclidean Geometry

Coordinate (Analytical) Geometry

November 2019

QUESTION 3

In the diagram, P, R(3 ; 5), S(−3 ; −7) and T(−5 ; k) are vertices of trapezium PRST and PT ∥ RS. RS and PR cut the y-axis at D and C(0 ; 5) respectively. PT and RS cut the x-axis at E and F respectively. $P\hat{E}F = \theta$.



3.1 Write down the equation of PR. (1)

3.2 Calculate the:

3.2.1 Gradient of RS (2)

3.2.2 Size of θ (3)

3.2.3 Coordinates of D (3)

3.3 If it is given that $TS = 2\sqrt{5}$, calculate the value of k. (4)

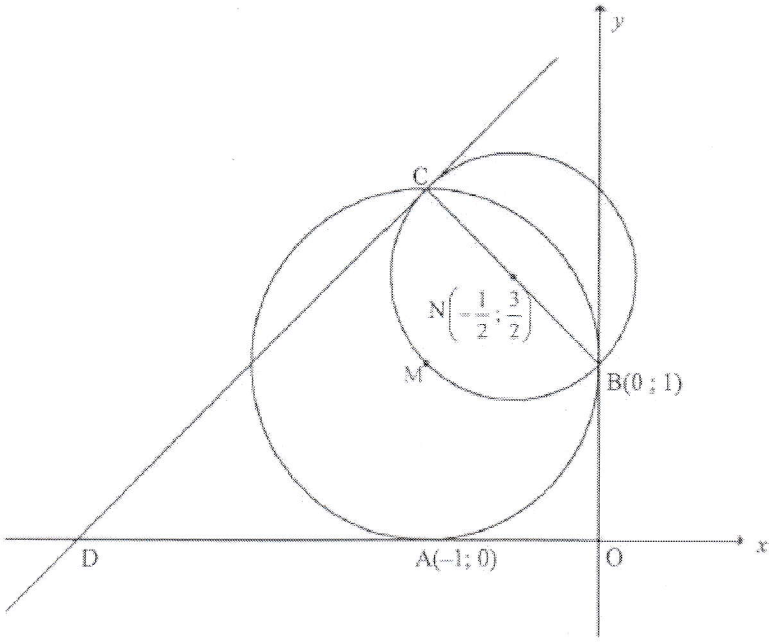
3.4 Parallelogram TDNS, with N in the 4^{th} quadrant, is drawn. Calculate the coordinates of N. (3)

3.5 ΔPRD is reflected about the y-axis to form $\Delta P'R'D'$. Calculate the size of $R\hat{D}R'$. (3)

[19]

QUESTION 4

In the diagram, a circle having centre M touches the x-axis at A(–1 ; 0) and the y-axis at B(0 ; 1). A smaller circle, centred at $N\left(-\frac{1}{2};\frac{3}{2}\right)$, passes through M and cuts the larger circle at B and C. BNC is a diameter of the smaller circle. A tangent drawn to the smaller circle at C, cuts the x-axis at D.



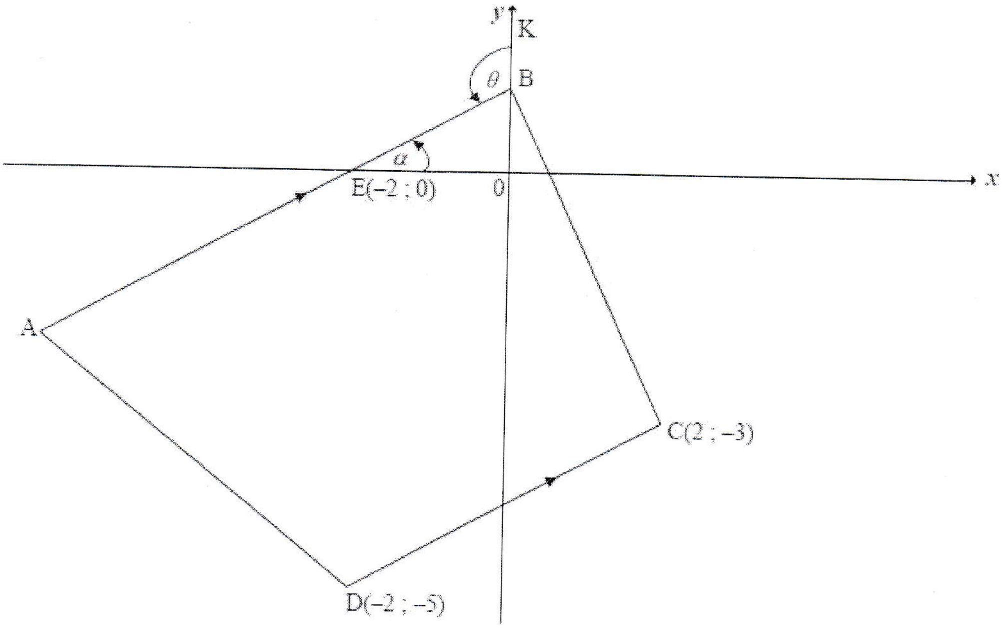
4.1	Determine the equation of the circle centred at M in the form $(x-a)^2+(y-b)^2=r^2$	(3)
4.2	Calculate the coordinates of C.	(2)
4.3	Show that the equation of the tangent CD is $y-x=3$.	(4)
4.4	Determine the values of t for which the line $y=x+t$ will NOT touch or cut the smaller circle.	(3)
4.5	The smaller circle centred at N is transformed such that point C is translated along the tangent to D. Calculate the coordinates of E, the new centre of the smaller circle.	(3)
4.6	If it is given that the area of quadrilateral OBCD is $2a^2$ square units and $a>0$, show that $a=\frac{\sqrt{7}}{2}$ units.	(5)

[20]

<u>May-June 2019</u>

QUESTION 3

In the diagram, A, B, C(2 ; −3) and D(−2 ; −5) are vertices of a trapezium with AB ∥ DC. E(−2 ; 0) is the x-intercept of AB. The inclination of AB is α. K lies on the y-axis and $K\hat{B}E = \theta$.



3.1 Determine:

3.1.1 The midpoint of EC (2)

3.1.2 The gradient of DC (2)

3.1.3 The equation of AB in the form $y = mx + c$ (3)

3.1.4 The size of θ (3)

3.2 Prove that AB ⊥ BC. (3)

3.3 The points E, B and C lie on the circumference of a circle. Determine:

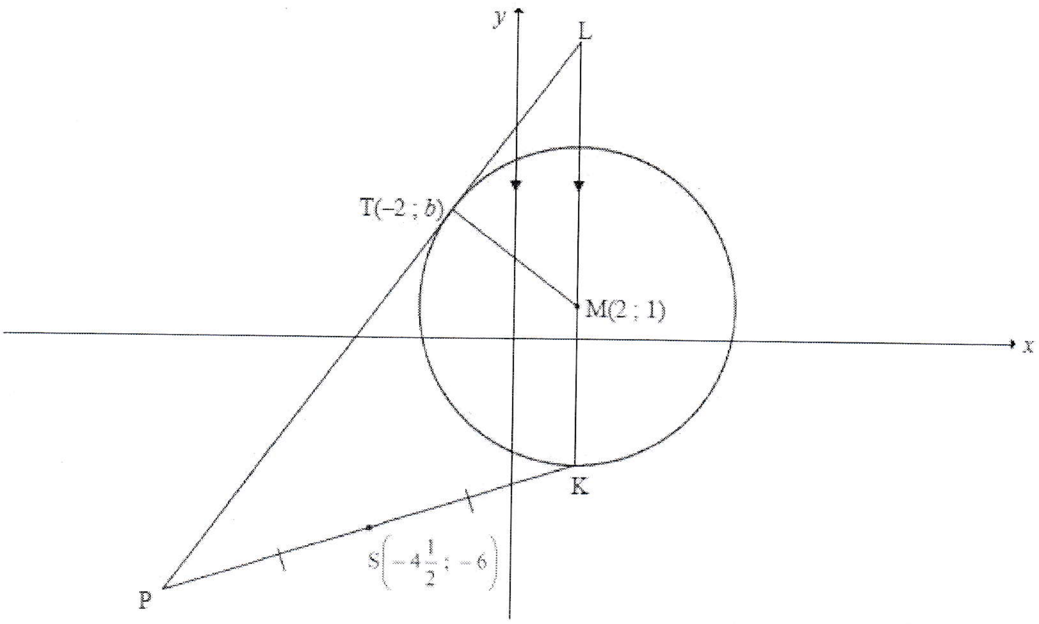
3.3.1 The centre of the circle (1)

3.3.2 The equation of the circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (4)

[18]

QUESTION 4

In the diagram, the circle is centred at M(2 ; 1). Radius KM is produced to L, a point outside the circle, such that KML ∥ y-axis. LTP is a tangent to the circle at T(–2 ; b). $S\left(-4\frac{1}{2}; -6\right)$ is the midpoint of PK.



4.1 Given that the radius of the circle is 5 units, show that $b = 4$. (4)

4.2 Determine:

4.2.1 The coordinates of K (2)

4.2.2 The equation of the tangent LTP in the form $y = mx + c$ (4)

4.2.3 The area of $\triangle$LPK (7)

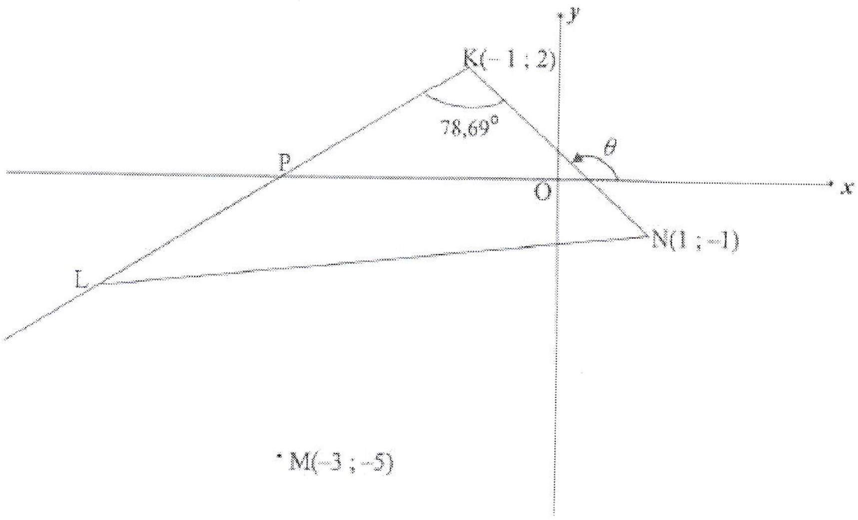
4.3 Another circle with equation $(x-2)^2 + (y-n)^2 = 25$ is drawn. Determine, with an explanation, the value(s) of n for which the two circles will touch each other externally. (4)

[21]

November 2018

QUESTION 3

In the diagram, K(– 1 ; 2), L and N(1 ; – 1) are vertices of ΔKLN such that $L\hat{K}N = 78{,}69°$. KL intersects the x-axis at P. KL is produced. The inclination of KN is θ. The coordinates of M are (–3 ; –5).



3.1 Calculate:

3.1.1 The gradient of KN (2)

3.1.2 The size of θ, the inclination of KN (2)

3.2 Show that the gradient of KL is equal to 1. (2)

3.3 Determine the equation of the straight line KL in the form $y = mx + c$. (2)

3.4 Calculate the length of KN. (2)

3.5 It is further given that KN = LM.

3.5.1 Calculate the possible coordinates of L. (5)

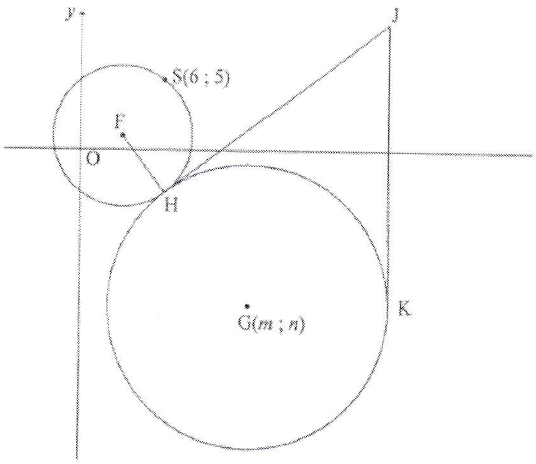
3.5.2 Determine the coordinates of L if it is given that KLMN is a parallelogram. (3)

3.6 T is a point on KL produced. TM is drawn such that TM = LM. Calculate the area of ΔKTN. (4)

[22]

QUESTION 4

In the diagram, the equation of the circle with centre F is $(x-3)^2+(y-1)^2=r^2$. S(6 ; 5) is a point on the circle with centre F. Another circle with centre G(m ; n) in the 4th quadrant touches the circle with centre F, at H such that FH : HG = 1 : 2. The point J lies in the first quadrant such that HJ is a common tangent to both these circles. JK is a tangent to the larger circle at K.



4.1 Write down the coordinates of F. (2)

4.2 Calculate the length of FS. (2)

4.3 Write down the length of HG. (1)

4.4 Give a reason why JH = JK. (1)

4.5 Determine:

4.5.1 The distance FJ, with reasons, if it is given that JK = 20 (4)

4.5.2 The equation of the circle with centre G in terms of m and n in the form $(x-a)^2+(y-b)^2=r^2$ (1)

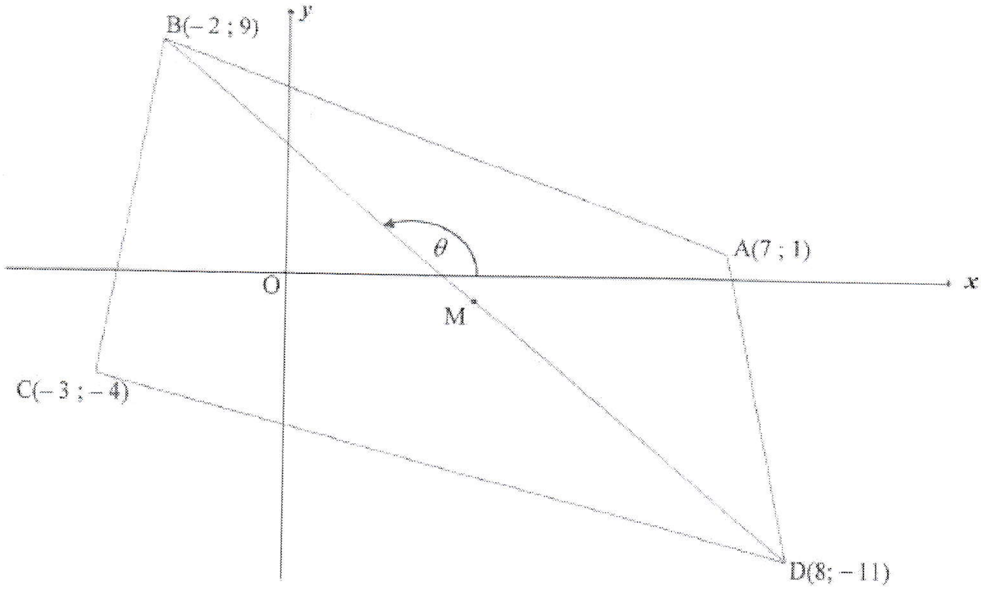
4.5.3 The coordinates of G, if it is further given that the equation of tangent JK is $x = 22$ (7)

[18]

June 2018

QUESTION 3

In the diagram, ABCD is a quadrilateral having vertices A(7 ; 1), B(– 2 ; 9), C(– 3 ; – 4) and D(8 ; – 11). M is the midpoint of BD.



3.1 Calculate the gradient of AC. (2)

3.2 Determine:

3.2.1 The equation of AC in the form $y = mx + c$ (2)

3.2.2 Whether M lies on AC (4)

3.3 Prove that BD ⊥ AC. (3)

3.4 Calculate:

3.4.1 θ, the inclination of BD (2)

3.4.2 The size of $C\hat{B}D$ (3)

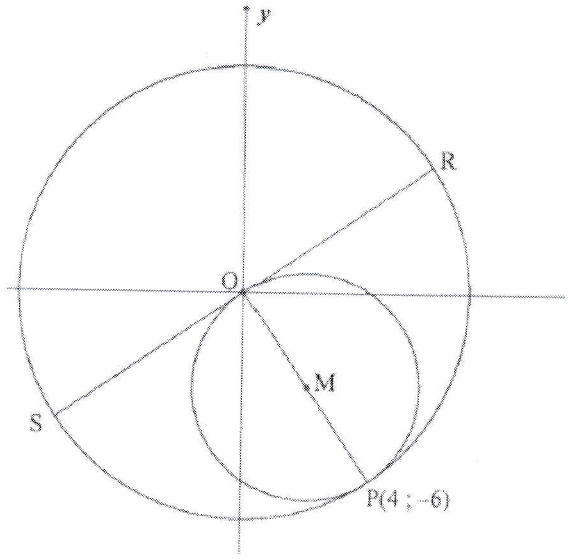
3.4.3 The length of AC (2)

3.4.4 The area of ABCD (5)

[23]

QUESTION 4

In the diagram, a circle having centre at the origin passes through P(4 ; – 6). PO is the diameter of a smaller circle having centre at M. The diameter RS of the larger circle is a tangent to the smaller circle at O.



4.1 Calculate the coordinates of M. (2)

4.2 Determine the equation of:

4.2.1 The large circle (2)

4.2.2 The small circle in the form $x^2 + y^2 + Cx + Dy + E = 0$ (3)

4.2.3 The equation of RS in the form $y = mx + c$ (3)

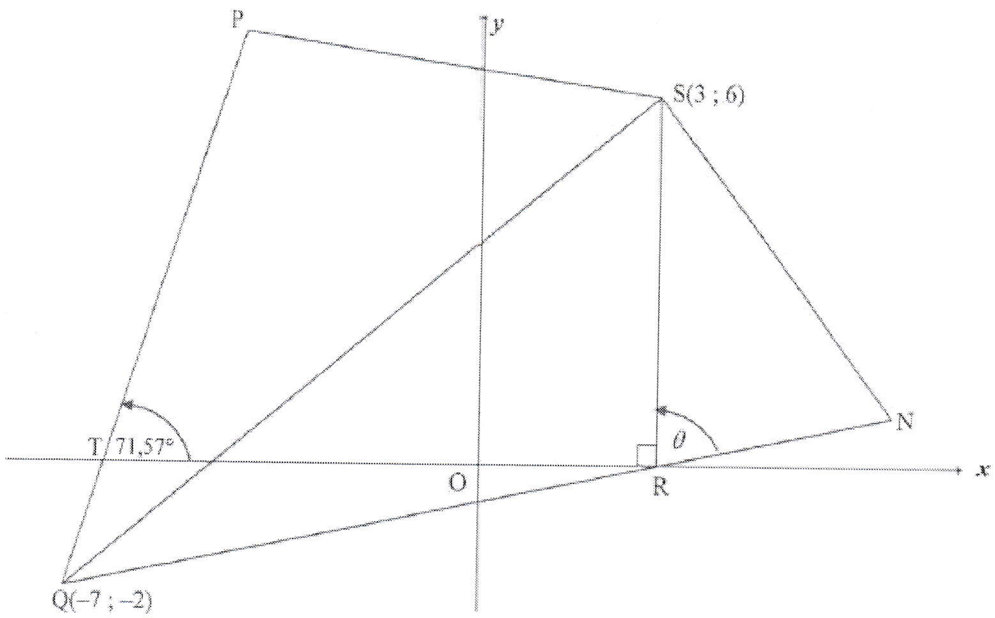
4.3 Determine the length of chord NR, where N is the reflection of R in the y-axis. (4)

4.4 The circle with centre at M is reflected about the x-axis to form another circle centred at K. Calculate the length of the common chord of these two circles. (3)
[17]

March 2018

QUESTION 3

In the diagram, P, Q(–7 ; –2), R and S(3 ; 6) are vertices of a quadrilateral. R is a point on the x-axis. QR is produced to N such that QR = 2RN. SN is drawn. $P\hat{T}O = 71{,}57°$ and $S\hat{R}N = \theta$.



Determine:

3.1 The equation of SR (1)

3.2 The gradient of QP to the nearest integer (2)

3.3 The equation of QP in the form $y = mx + c$ (2)

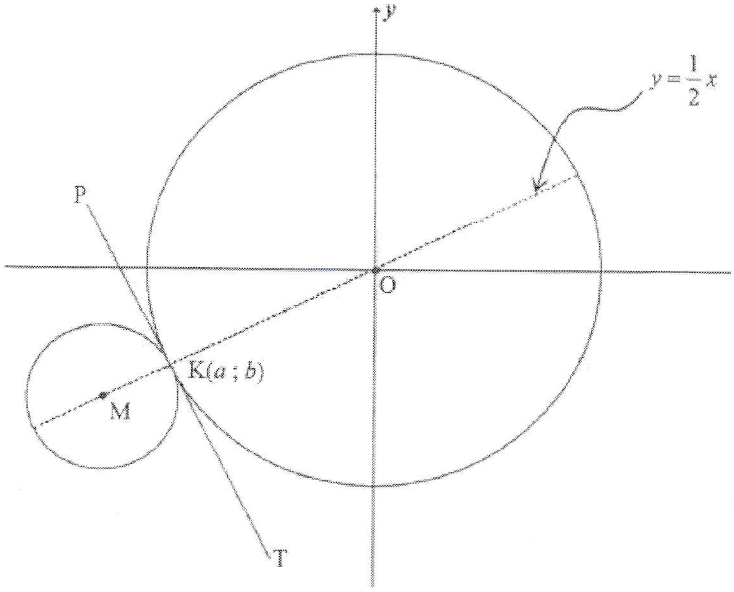
3.4 The length of QR. Leave your answer **in surd form**. (2)

3.5 $\tan(90° - \theta)$ (3)

3.6 The area of ΔRSN, **without using a calculator** (6)
[16]

QUESTION 4

In the diagram, PKT is a common tangent to both circles at K(a ; b). The centres of both circles lie on the line $y = \frac{1}{2}x$. The equation of the circle centred at O is $x^2 + y^2 = 180$. The radius of the circle is three times that of the circle centred at M.



4.1 Write down the length of OK **in surd form**. (1)

4.2 Show that K is the point (–12 ; –6). (4)

4.3 Determine:

4.3.1 The equation of the common tangent, PKT, in the form $y = mx + c$ (3)

4.3.2 The coordinates of M (6)

4.3.3 The equation of the smaller circle in the form $(x-a)^2 + (y-b)^2 = r^2$ (2)

4.4 For which value(s) of r will another circle, with equation $x^2 + y^2 = r^2$, intersect the circle centred at M at two distinct points? (3)

4.5 Another circle, $x^2 + y^2 + 32x + 16y + 240 = 0$, is drawn. Prove by calculation that this circle does NOT cut the circle with centre M(–16 ; –8). (5)

[24]

November 2017: