<u>**APPLICATION OF CALCULUS**</u>

<u>**May – June 2021**</u>

QUESTION 10

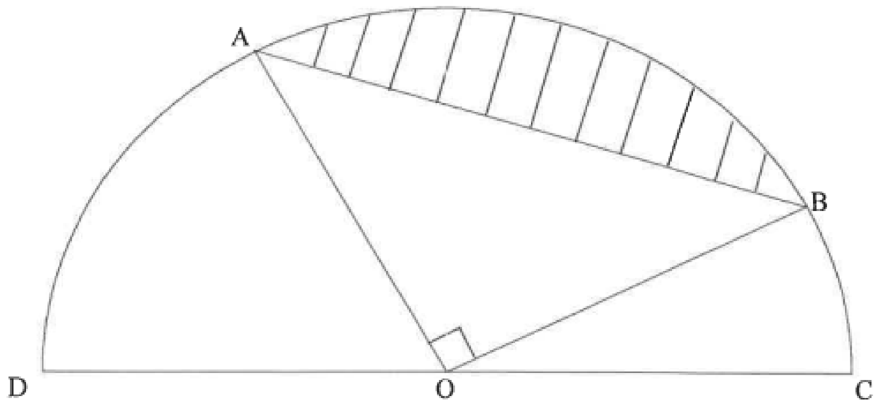
10.1 The graph of $f(x) = ax^3 + bx^2 + cx + d$ has two turning points.

The following information about f is also given:

- $f(2) = 0$
- The x-axis is a tangent to the graph of f at $x = -1$
- $f'(1) = 0$
- $f'\left(\frac{1}{2}\right) > 0$

Without calculating the equation of f, use this information to draw a sketch graph of f, only indicating the x-coordinates of the x-intercepts and turning points. (4)

10.2 O is the centre of a semicircle passing through A, B, C and D. The radius of the semicircle is $\left(x - x^2\right)$ units for $0 < x < 1$. ΔAOB is right-angled at O.



10.2.1 Show that the area of the shaded part is given by:

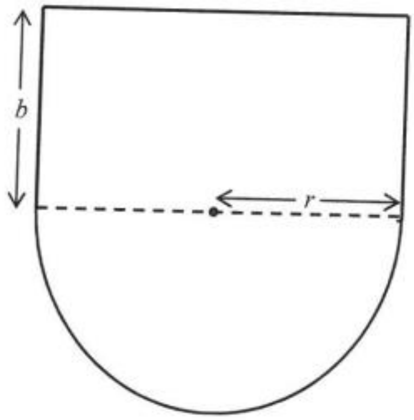
$$\text{Area} = \left(\frac{\pi - 2}{4}\right)\left(x^4 - 2x^3 + x^2\right)$$ (5)

10.2.2 Determine the value of x for which the shaded area will be a maximum. (4)

[13]

May – June 2017

QUESTION 10



The figure above shows the design of a theatre stage which is in the shape of a semicircle attached to a rectangle. The semicircle has a radius r and the rectangle has a breadth b. The perimeter of the stage is 60 m.

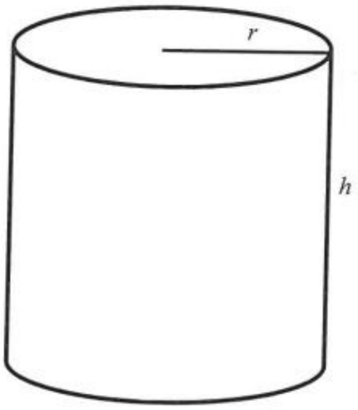
10.1 Determine an expression for b in terms of r. (2)

10.2 For which value of r will the area of the stage be a maximum? (6)

[8]

QUESTION 9

A 340 mℓ can with height h cm and radius r cm is shown below.



1 mℓ = 1 cm^3

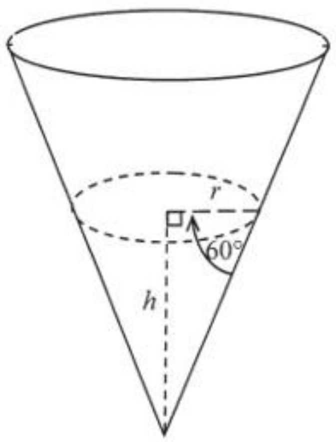
9.1 Determine the height of the can in terms of the radius r. (3)

9.2 Calculate the length of the radius of the can, in cm, if the surface area is to be a minimum. (6)

[9]

QUESTION 10

A rain gauge is in the shape of a cone. Water flows into the gauge. The height of the water is h cm when the radius is r cm. The angle between the cone edge and the radius is 60°, as shown in the diagram below.



Formulae for volume:

$V = \pi r^2 h$ $\quad V = \frac{1}{3}\pi r^2 h$

$V = lbh$ $\quad V = \frac{4}{3}\pi r^3$

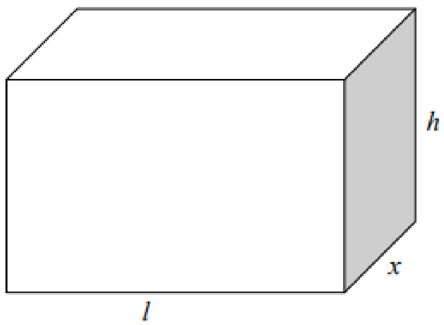
10.1 Determine r in terms of h. Leave your answer in surd form. (2)

10.2 Determine the derivative of the volume of water with respect to h when h is equal to 9 cm. (5)

[7]

QUESTION 11

A rectangular box is constructed in such a way that the length (l) of the base is three times as long as its width. The material used to construct the top and the bottom of the box costs R100 per square metre. The material used to construct the sides of the box costs R50 per square metre. The box must have a volume of 9 m^3. Let the width of the box be x metres.



11.1 Determine an expression for the height (h) of the box in terms of x. (3)

11.2 Show that the cost to construct the box can be expressed as $C = \frac{1200}{x} + 600x^2$. (3)

11.3 Calculate the width of the box (that is the value of x) if the cost is to be a minimum. (4)

[10]

NOV 2019

QUESTION 8

After flying a short distance, an insect came to rest on a wall. Thereafter the insect started crawling on the wall. The path that the insect crawled can be described by $h(t) = (t-6)(-2t^2 + 3t - 6)$, where h is the height (in cm) above the floor and t is the time (in minutes) since the insect started crawling.

8.1 At what height above the floor did the insect start to crawl? (1)

8.2 How many times did the insect reach the floor? (3)

8.3 Determine the maximum height that the insect reached above the floor. (4)
[8]

Given: $f(x) = 3x^3$

9.1 Solve $f(x) = f'(x)$ (3)

9.2 The graphs f, f' and f'' all pass through the point $(0 ; 0)$.

9.2.1 For which of the graphs will $(0 ; 0)$ be a stationary point? (1)

9.2.2 Explain the difference, if any, in the stationary points referred to in QUESTION 9.2.1. (2)

9.3 Determine the vertical distance between the graphs of f' and f'' at $x = 1$. (3)

9.4 For which value(s) of x is $f(x) - f'(x) < 0$? (4)
[13]